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Bayesian Nonparametric Nonlinear System Identification

A quick overview

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Two Different Approaches to Modelling

Maximum Likelihood (or PEM) + Regularisation

- Optimisation to find point estimates of parameters given the data.

$$\theta^* = \arg\min_{\theta} L(\mathcal{D}, \theta) + J(\theta)$$

$$x_{t+1} = f(x_t, \theta^*)$$

Bayesian

- Averaging over parameter distributions to find distributions over predictions

$$p(x_{t+1} \mid x_t, \mathcal{D}) = \int f(x_{t+1} \mid x_t, \theta) \; p(\theta \mid \mathcal{D}) \; d\theta$$

Why Bayesian?

- *No overfitting* because there is no fitting!
- No need to artificially limit the complexity of the models.
- Even with a finite set of *perfect observations* we may still be uncertain about our model/parameters.

Why Nonparametric?

In a parametric model

$$p(x_{t+1} \mid x_t, \theta, \mathcal{D}) = p(x_{t+1} \mid x_t, \theta)$$

In a nonparametric model, we perform inference on the space of functions, not parameters!

- Data is not condensed in a finite set of parameters.
- Flexibility not constrained by choice of parametric form.
- Nonparametric model can be very complex if the data supports this complexity.

Parallels between Bayesian Approach and Regularisation

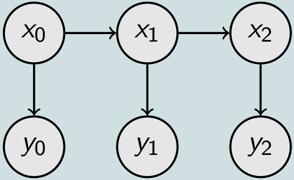
- Prior $\sim$ Regulariser.
- Inductive bias. There is no inference without assumptions
- One should not be afraid of priors: they are a very honest way to make assumptions that in other methods may be hidden inside algorithms.

Generative Models

A model that describes the data that can be observed from the system. It allows us to:

- Generate "fantasies" invented by the model.
- *Condition* on actual observations and infer the latent quantities.

$$
\begin{aligned}
p(a) &= \mathrm{Uniform}(-1, 1) \\
p(x_0) &= \mathcal{N}(0, 1) \\
p(x_{t+1}) &= \mathcal{N}(a x_t, 1) \\
p(y_t) &= \mathcal{N}(x_t, 1)
\end{aligned}
$$



Bayesian Nonparametric NARX Models

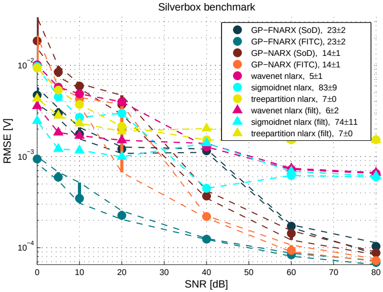
Take a NARX model

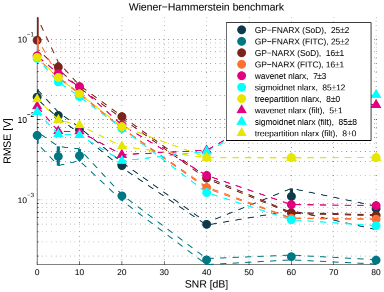
$$y_t = f(y_{t-1}, y_{t-2}, \cdots, u_{t-1}, \cdots)$$

and learn f in a nonparametric fashion, e.g. putting a Gaussian process prior over it.
Problem: this is an errors-in-variables regression since the inputs to f are noisy.

System Identification with Bayesian Nonparametric NARX Models with Filtered Regressors [1]

- Treating the model in a probabilistically consistent way is hard.
- Pragmatic solution: pre-process signals to reduce noise and put it as a regression problem.





[1] work with Carl E. Rasmussen.

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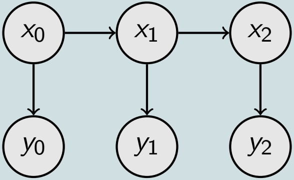
Bayesian Nonparametric Nonlinear State-Space Models

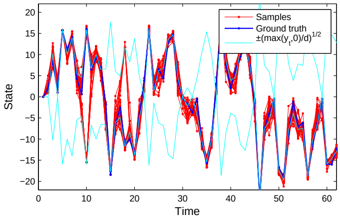
Nonparametric model for the state transition. Problem: states are not observed so this is not straightforward regression.

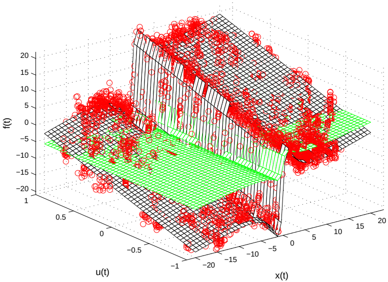


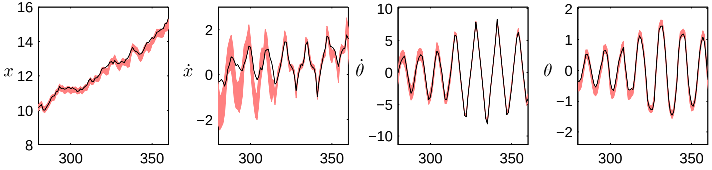
Example:
(this is the true, but unknown to us, system description)

$$
\begin{aligned}
x_{t+1} &= ax_t + bx_t/(1+x_t^2) + cu_t + v_t, \quad v_t \sim \mathcal{N}(0,q) \\
y_t &= dx_t^2 + e_t, \quad e_t \sim \mathcal{N}(0,r)
\end{aligned}
$$

Sys. Id. with Bayesian Nonparametric State-Space Models using Gaussian Processes and Particle MCMC [2]







[2] work with Fredrik Lindsten, Thomas B. Schön and Carl E. Rasmussen.

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