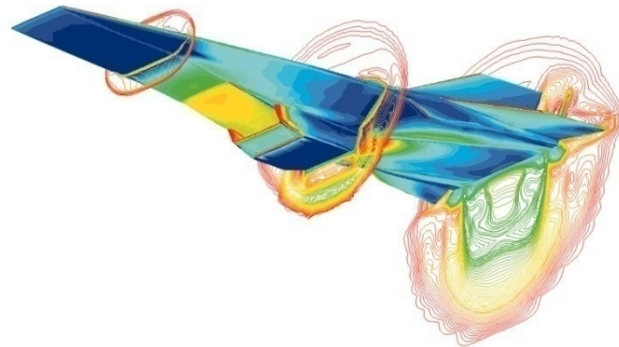


# Engineering in the Age of Machine Learning

Barcelona, 30 October 2015

Roger Frigola



# Machine Learning

- Create models based on data to make inferences/predictions.
- Based on statistics and computing.
- Reality is noisy and uncertain.

# Engineering

- Create realities that had never existed before.
- Based on physics.
- Reality is mostly deterministic.

# My Background

- Degrees in **General and Aerospace Engineering**
- AIRBUS (1.5 years) Fluid-Control-Structure Interaction
- McLaren F1 Team (4 years) Simulation, Modelling, Optimisation,
- PhD in **Machine Learning**
- Consulting at Ferrari F1, Red Bull F1, NZ America's Cup team

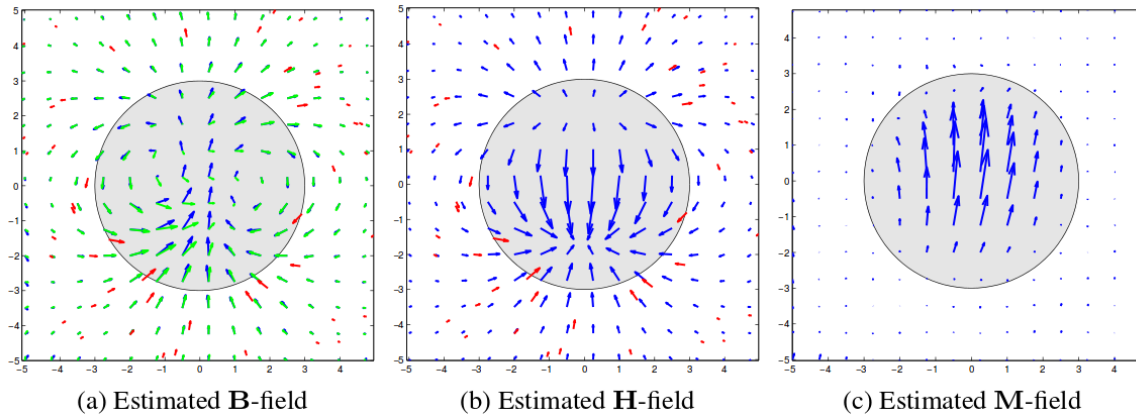


# Outline

- Model-based Machine Learning
- Bayesian Inference
- Experiment Design and Optimisation
- The Future?

# Model-Based Machine Learning

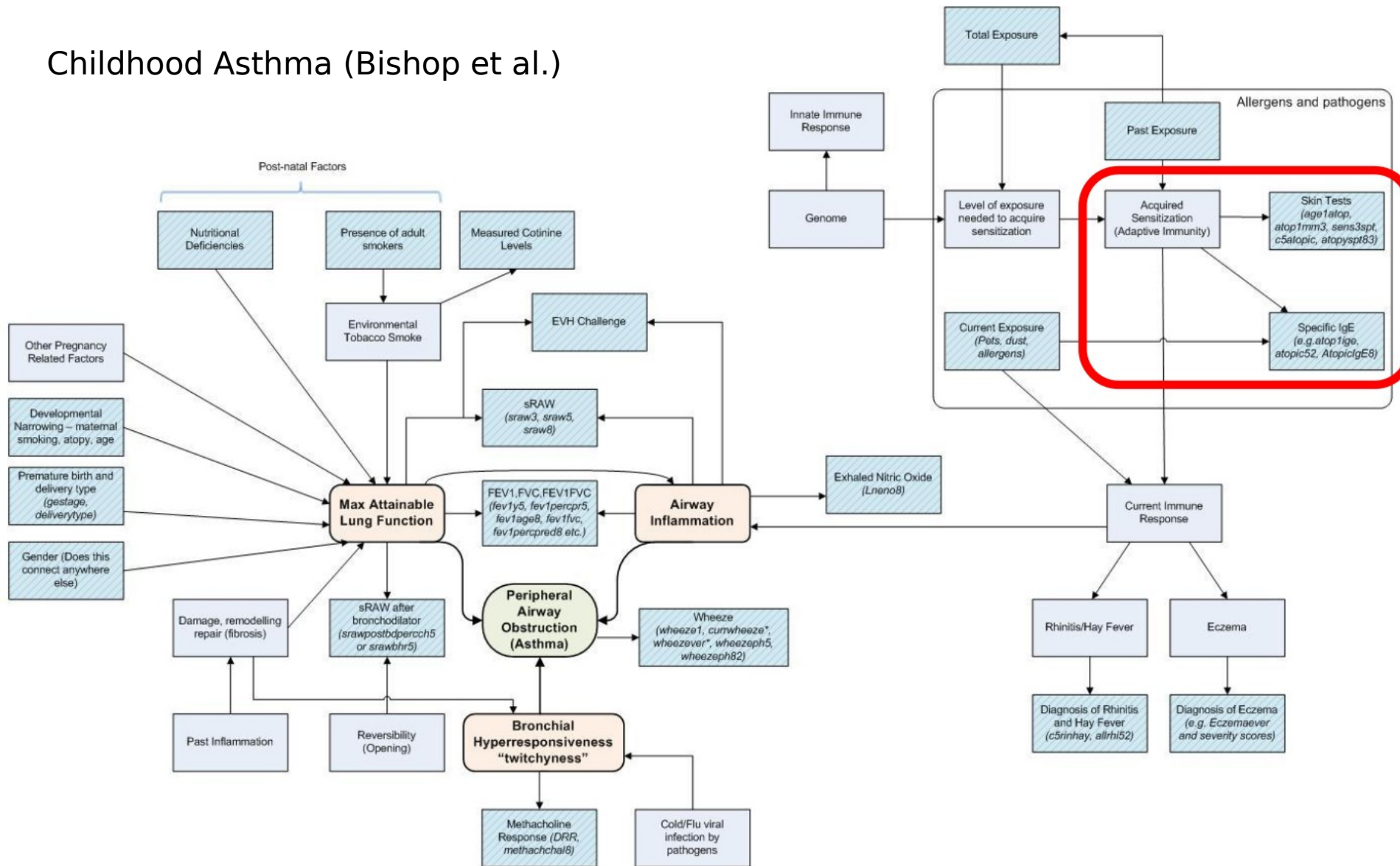
- Don't rely only on data. Use the knowledge we have about the world!



Make customised assumptions!

# Model-Based Machine Learning

## Childhood Asthma (Bishop et al.)



# Bayesian Inference for Engineering. Why?



# Introduction to Bayesian Modelling and Inference

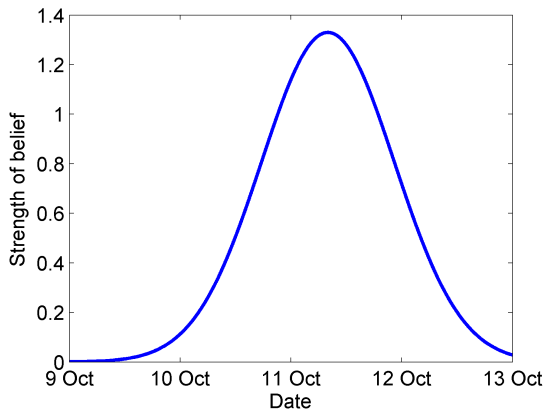
Uses probability to *quantify uncertainty*.

Related to information rather than repeated trials.

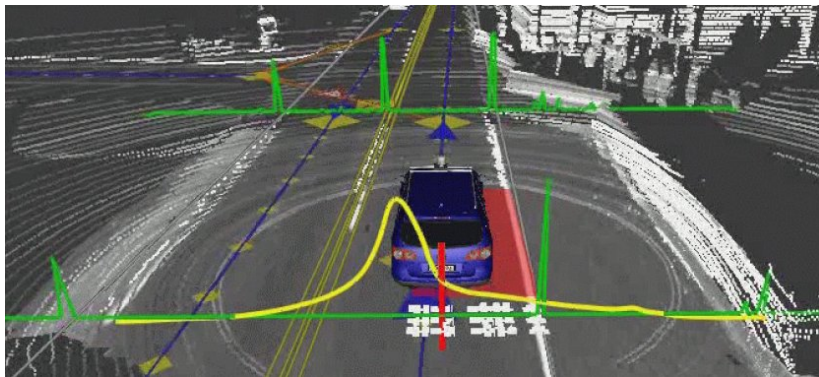
Uncertainty is subjective, it depends on what we have seen.



# Subjective Uncertainty



# Subjective Uncertainty



Stanford's self-driving car for the DARPA Urban Challenge (2007).

# Bayesian Inference of Power and Drag



Infer power and drag based on noisy *acceleration measurements* using a simple inertial and aerodynamic model

$$a_t = \frac{1}{mV_t} P - \frac{\rho V_t^2 S}{2m} C_d$$

e.g. with Gaussian noise

$$y_t = \mathcal{N}(a_t, \sigma^2)$$

$$\mathcal{D} = \{y_1, \dots, y_N, V_1, \dots, V_N\}$$

# Bayesian Inference of Power and Drag



Infer power and drag based on noisy *acceleration measurements* using a simple inertial and aerodynamic model

GOAL: find probability distribution of unknown parameters given data

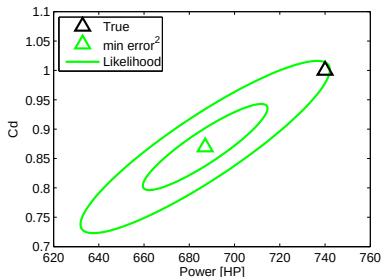
$$p(\theta \mid \mathcal{D}) = \frac{p(\mathcal{D} \mid \theta) p(\theta)}{p(\mathcal{D})}$$

$$p(\theta \mid \mathcal{D}) \propto p(\mathcal{D} \mid \theta) p(\theta)$$

# Bayesian Inference of Power and Drag



Infer power and drag based on noisy *acceleration measurements* using a simple inertial and aerodynamic model

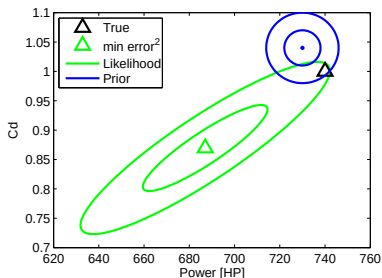


$$p(\mathcal{D} \mid \theta)$$

# Bayesian Inference of Power and Drag



Infer power and drag based on noisy *acceleration measurements* using a simple inertial and aerodynamic model

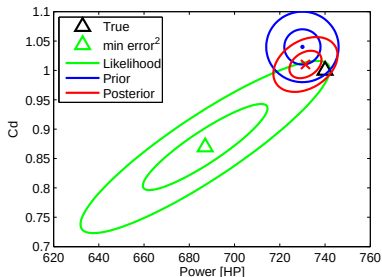


$p(\theta)$

# Bayesian Inference of Power and Drag



Infer power and drag based on noisy *acceleration measurements* using a simple inertial and aerodynamic model



$$p(\theta \mid \mathcal{D})$$

# Bayesian Inference

One could say that we have used the prior as a regulariser to solve

$$\theta^* = \arg \min_{\theta} L(\theta, \mathcal{D}) + J(\theta)$$

But, we were simply looking for the posterior:  $p(\theta \mid \mathcal{D})$ .

*No optimisation!*

The posterior represents our uncertainty and tells us how to average different models.



# Bayesian Inference

What is the outcome of Bayesian inference?

Thomas' indoor localisation example.

Posterior over parameters  $\rightarrow$  posterior over identified systems.

In fact, we can find posteriors over many different kinds of objects: functions, genetic trees, English language sentences, etc.

# Bayesian Inference: Making Predictions

The posterior represents our uncertainty over the parameters.

Any prediction can be found by *averaging* over the posterior

$$\begin{aligned} p(\text{LapTime} \mid \mathcal{D}) &= \int p(\text{LapTime}, \theta \mid \mathcal{D}) d\theta \\ &= \int p(\text{LapTime} \mid \theta) p(\theta \mid \mathcal{D}) d\theta. \end{aligned}$$

# Making Decisions Under Uncertainty

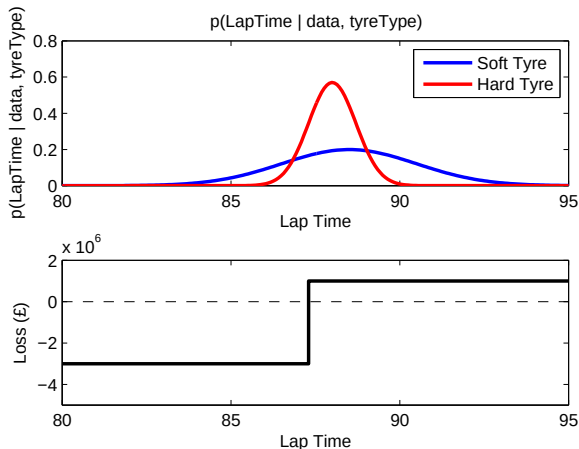
Bayesian inference provides probability distributions.

But, often, we can only take one action.

One solution: take the action that minimises the expected loss (aka risk) under the uncertainty provided by Bayesian inference.

$$\mathbf{a}_{\text{opt}} = \arg \min_a \int \text{Loss}(\mathbf{a}, \theta) p(\theta \mid \mathcal{D}) d\theta$$

# Making Decisions Under Uncertainty



Expected loss for hard tyre:  $\$3.65 \cdot 10^5$

Expected loss for soft tyre:  $\$-0.98 \cdot 10^5$

# Over-fitting and Counting Parameters

Bayesian methods do not overfit because there is *no fitting!*

Inference is based on integration, i.e. averaging.

There is no statistical price to pay for adding more parameters.

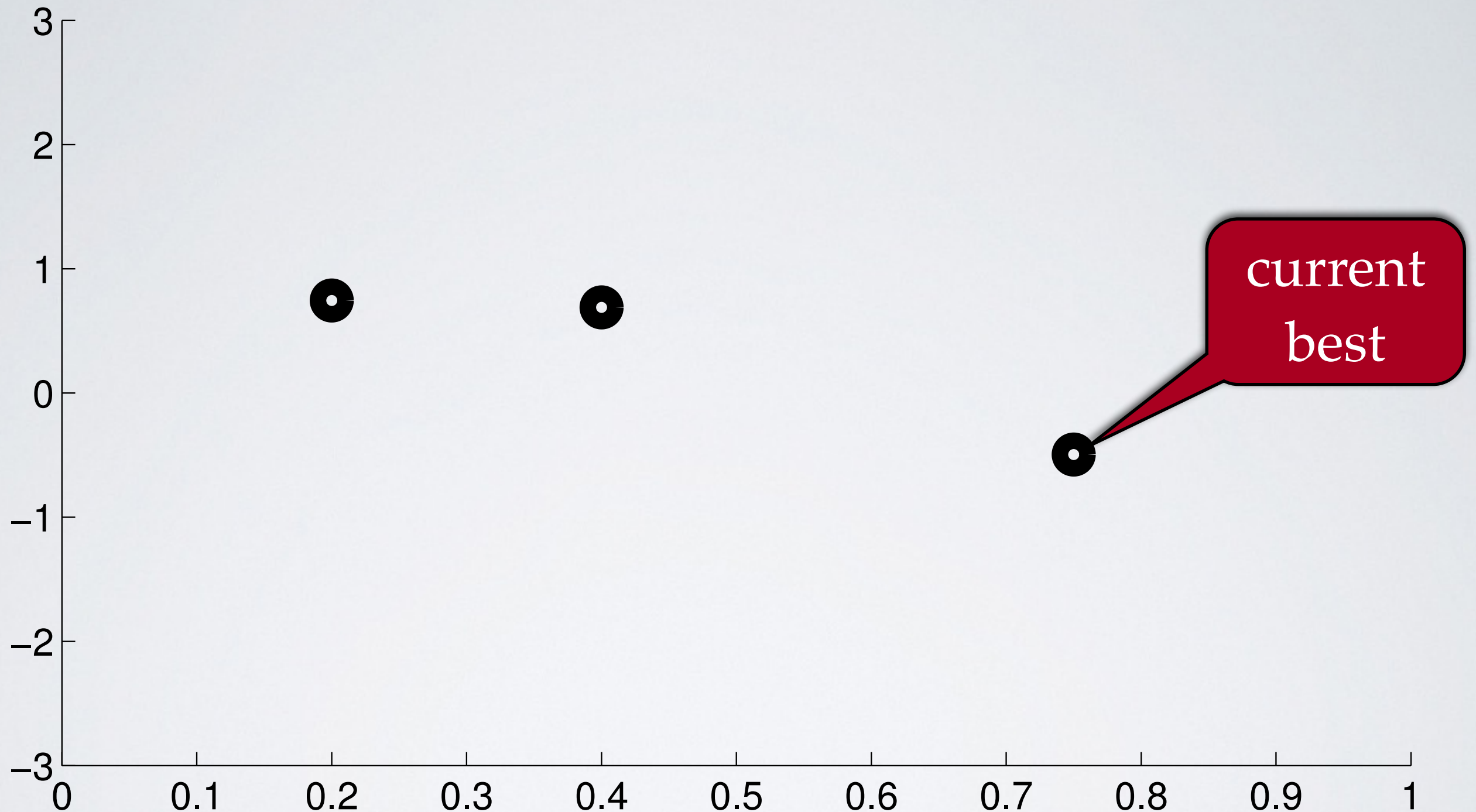
Nonidentifiability is not a problem when making predictions.

# Experiment Design and Optimisation

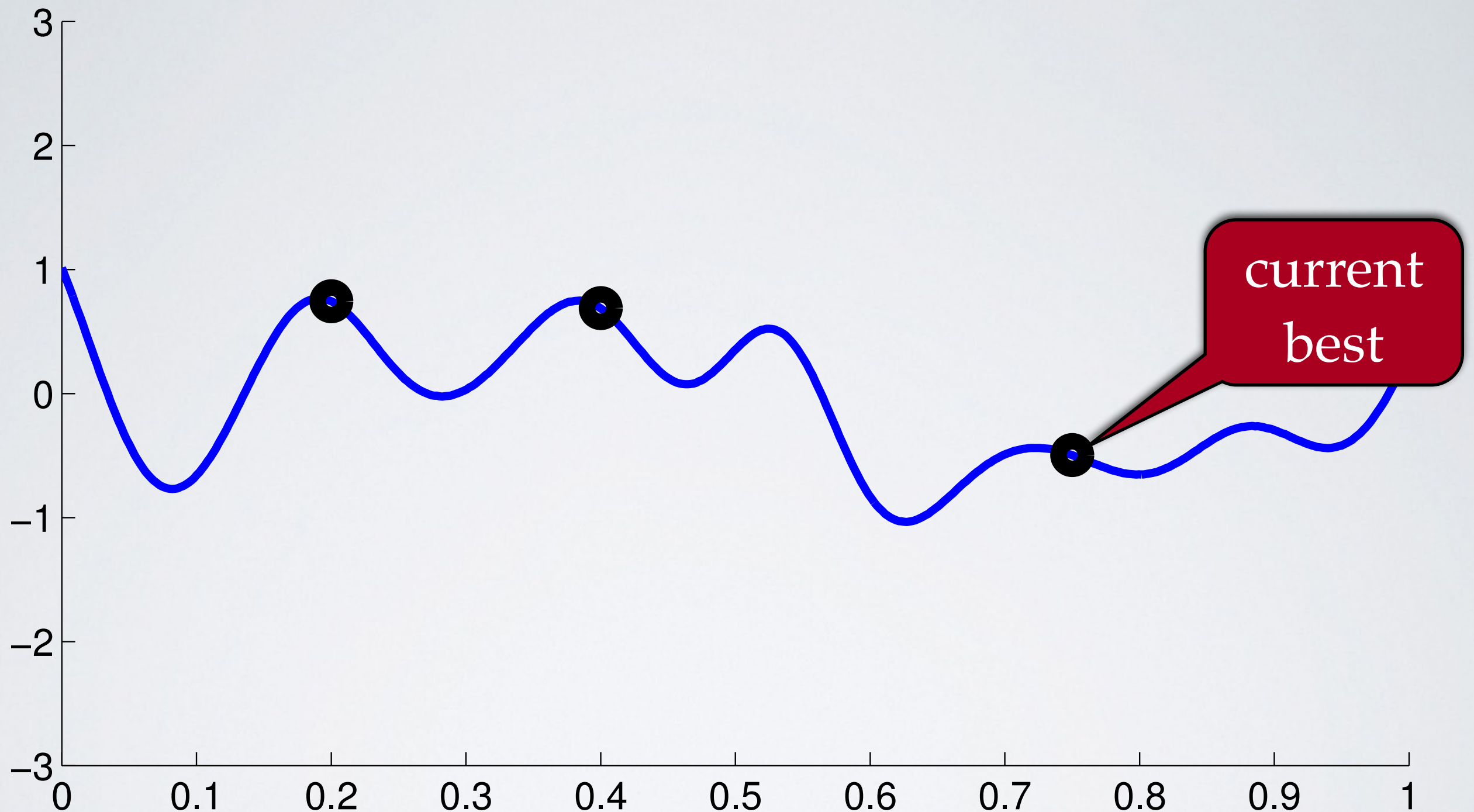
- In engineering we can run simulations or make prototypes.
  - Which simulations to run? What prototypes to build?
  - What is our goal?
- 
- Following slides from Ryan Adams, A Tutorial on Bayesian Optimization for Machine Learning (2014).

# The Bayesian Optimization Idea

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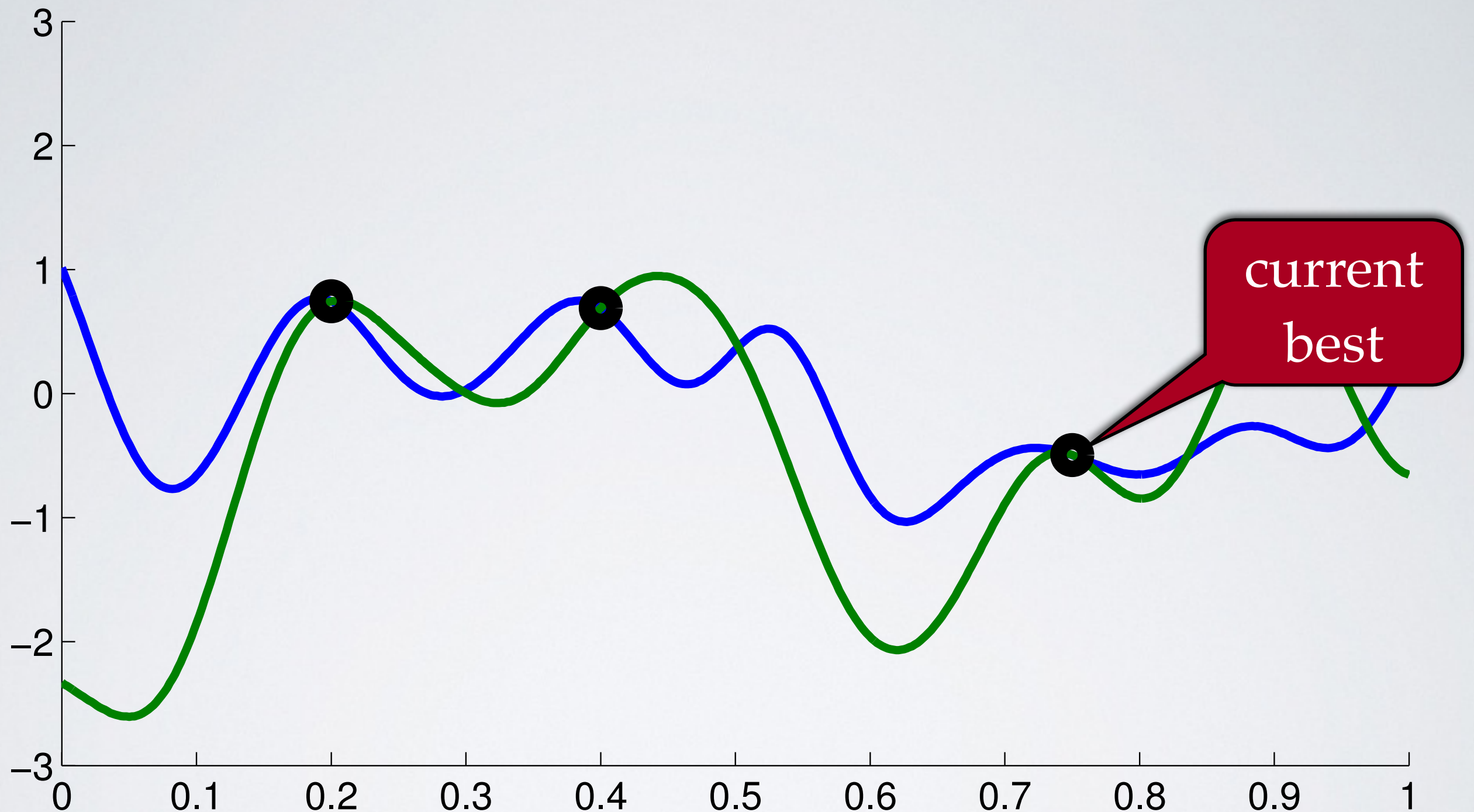


# The Bayesian Optimization Idea

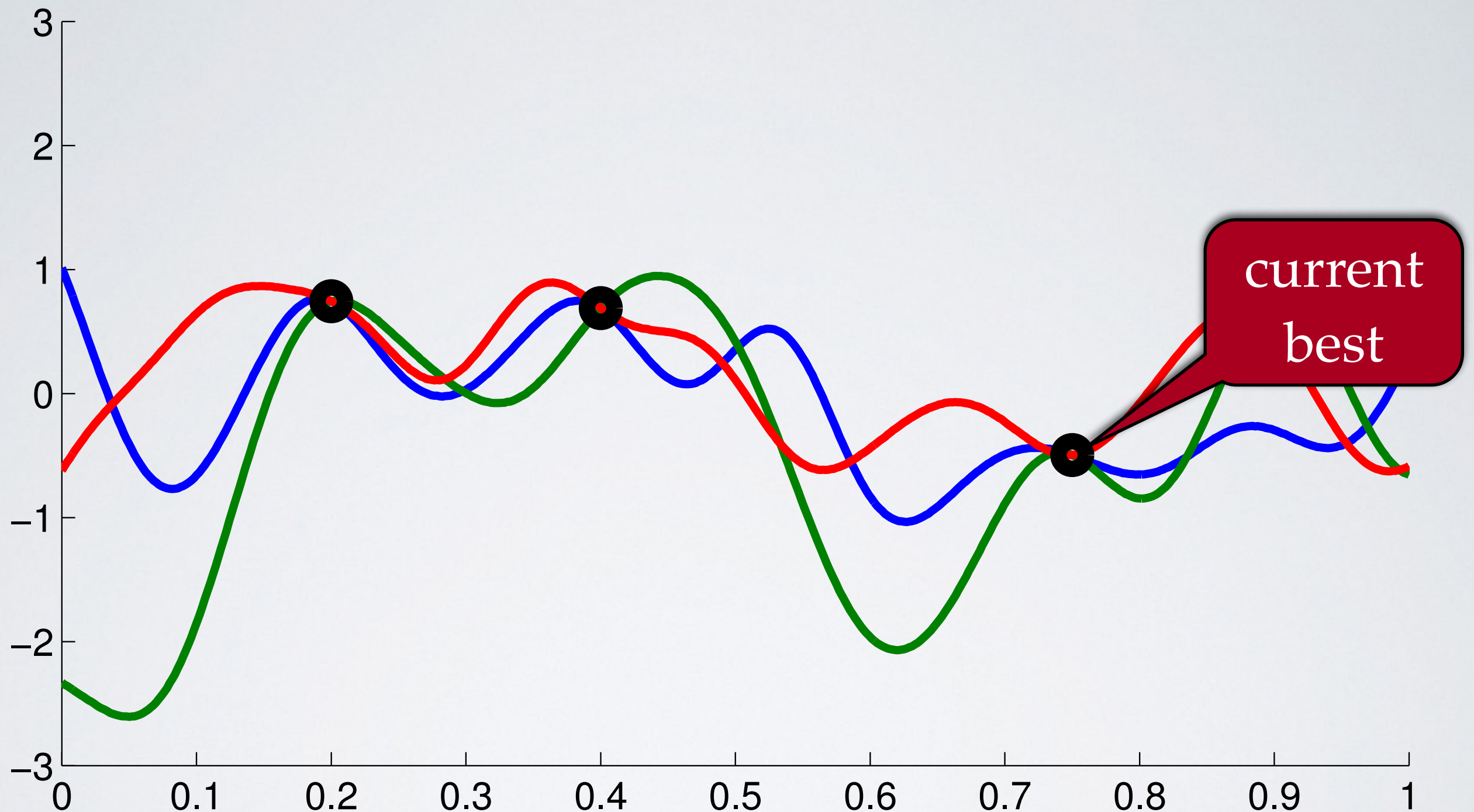




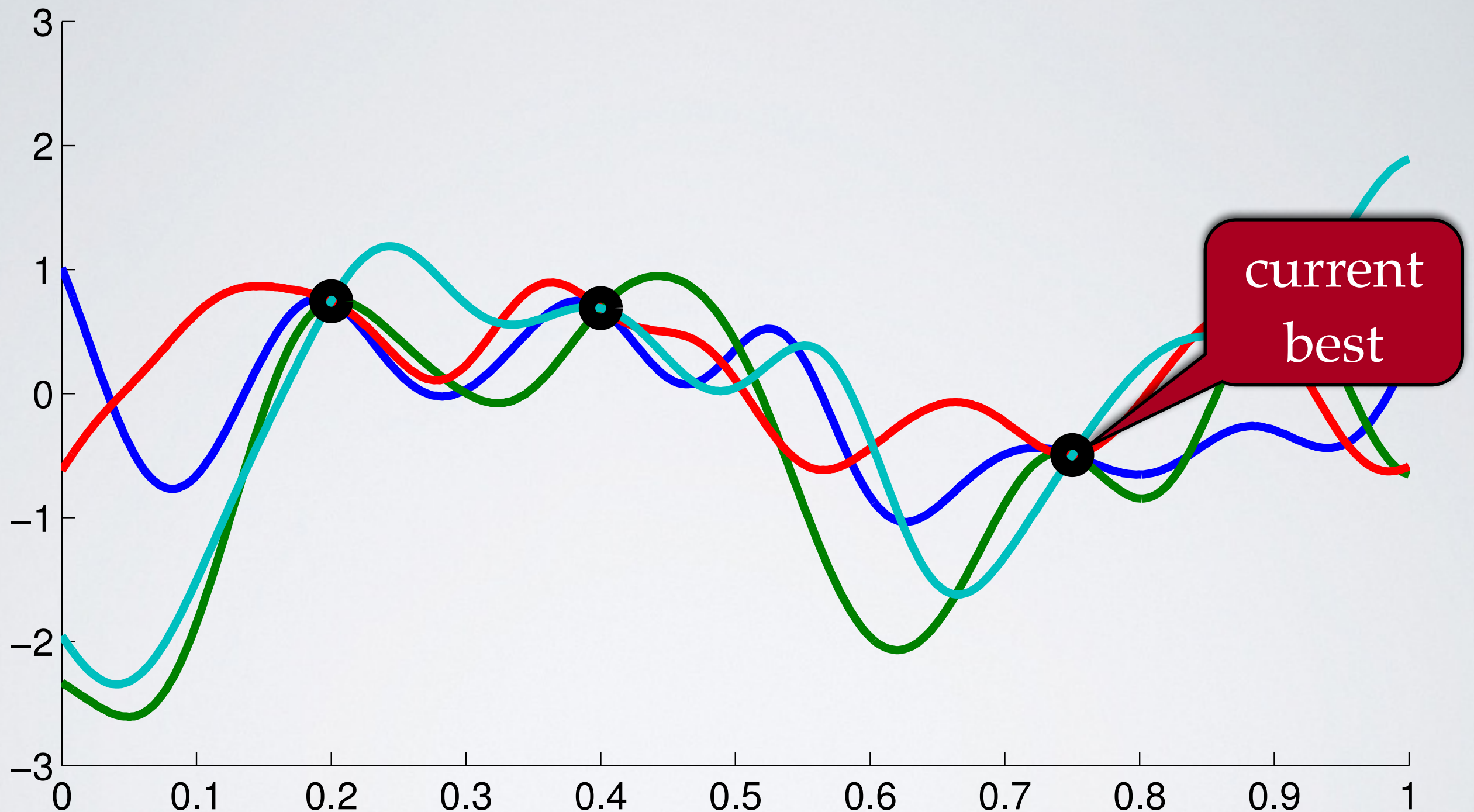
# The Bayesian Optimization Idea



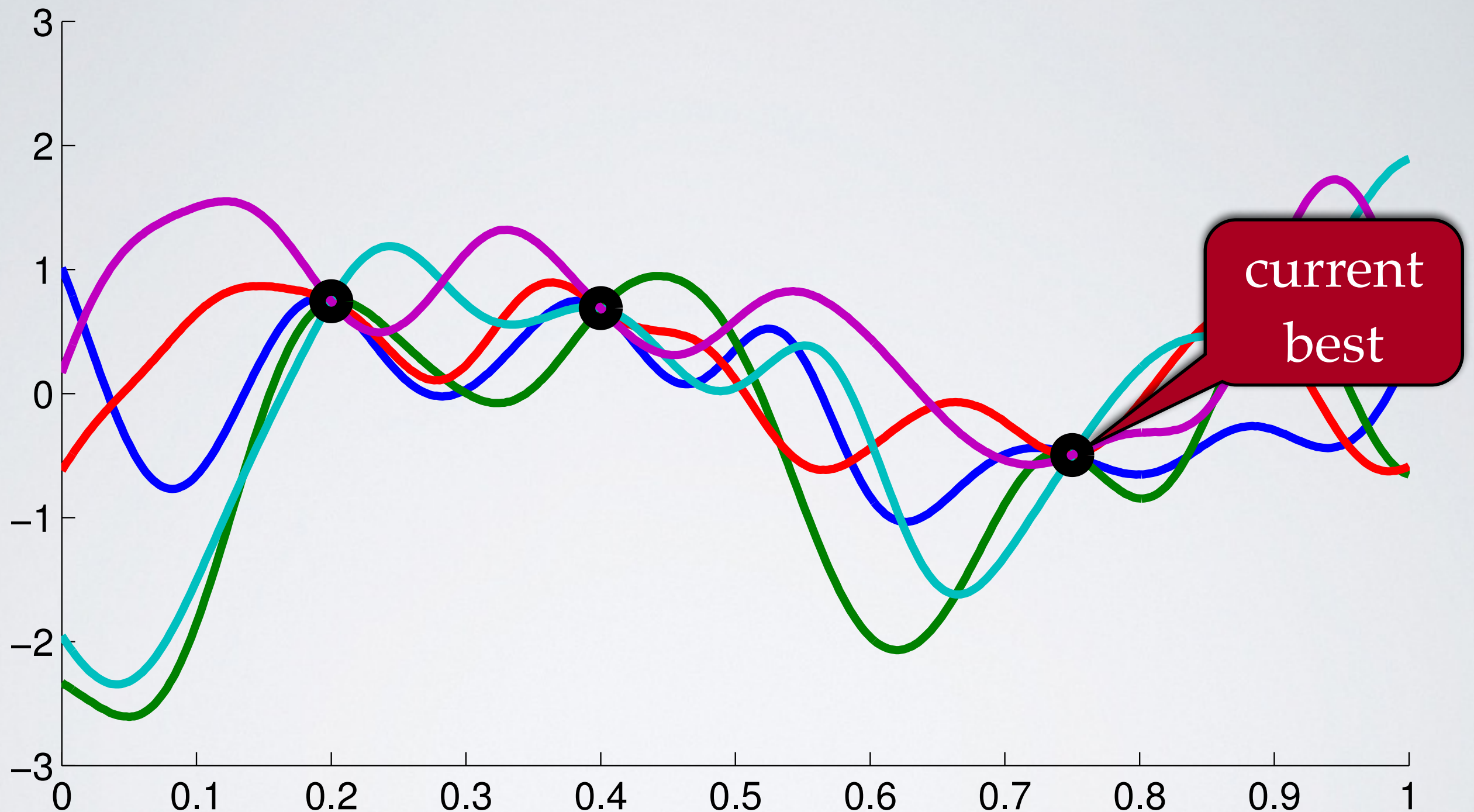
# The Bayesian Optimization Idea



# The Bayesian Optimization Idea

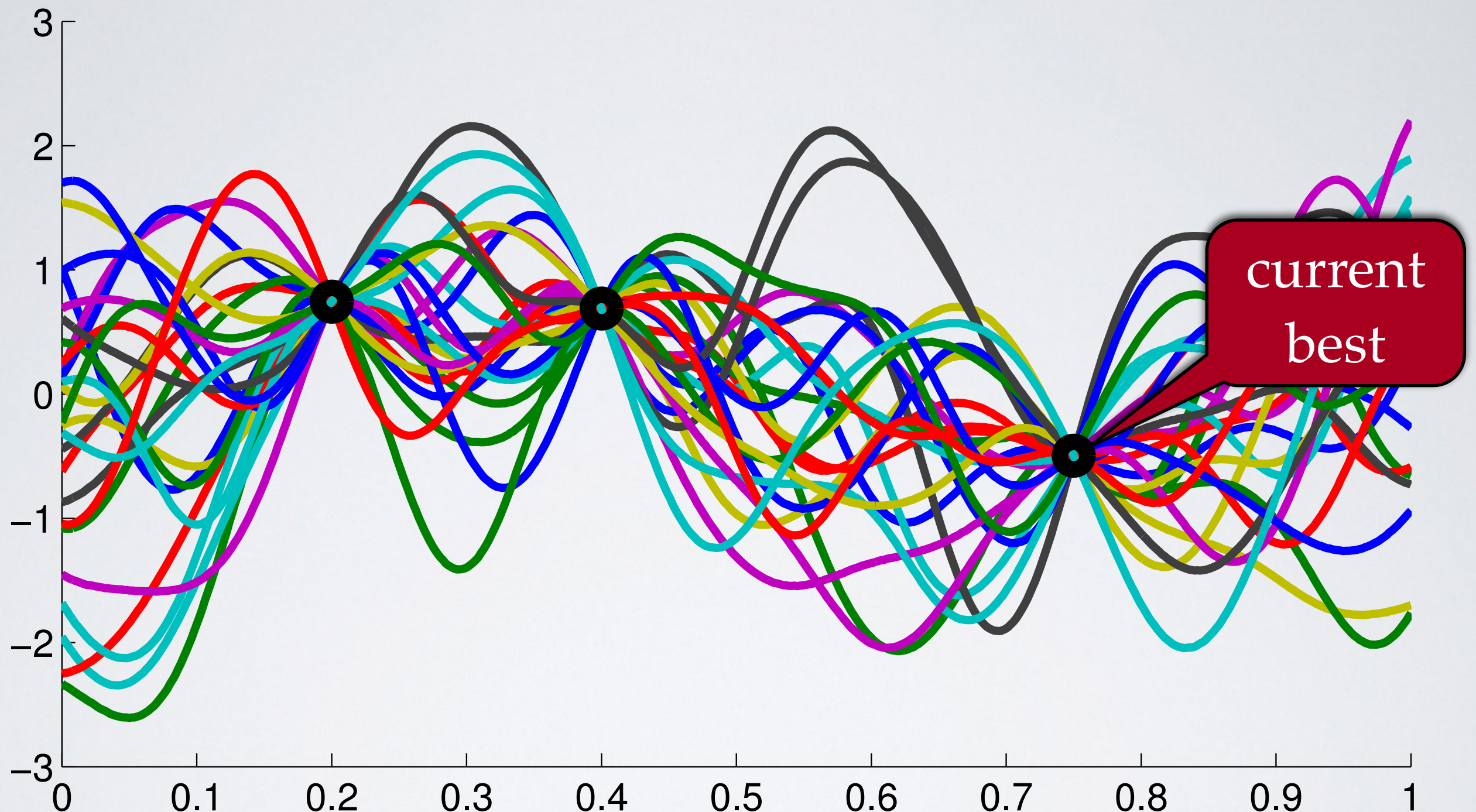


# The Bayesian Optimization Idea



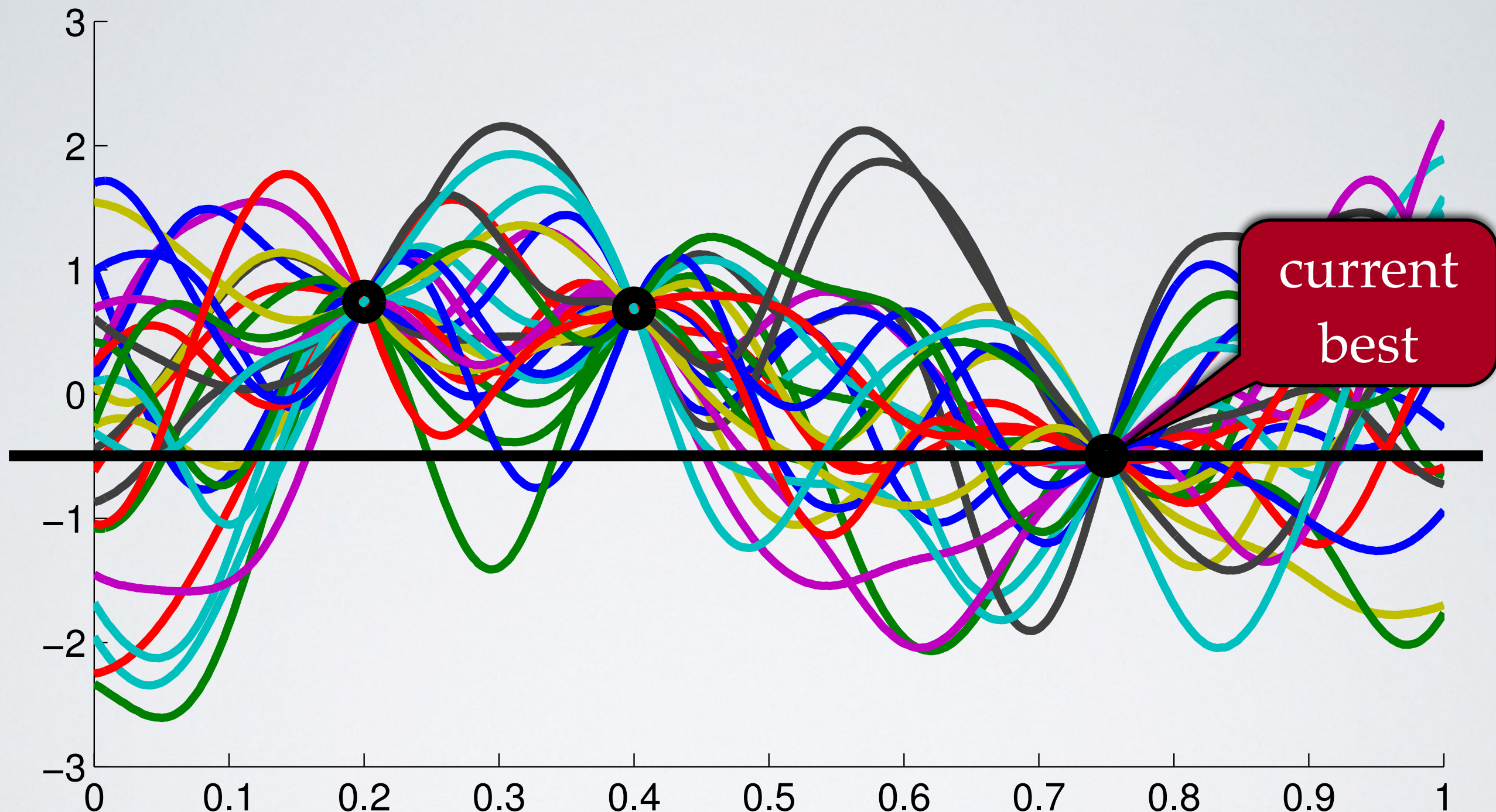


# The Bayesian Optimization Idea



Where should we evaluate next  
in order to improve the most?

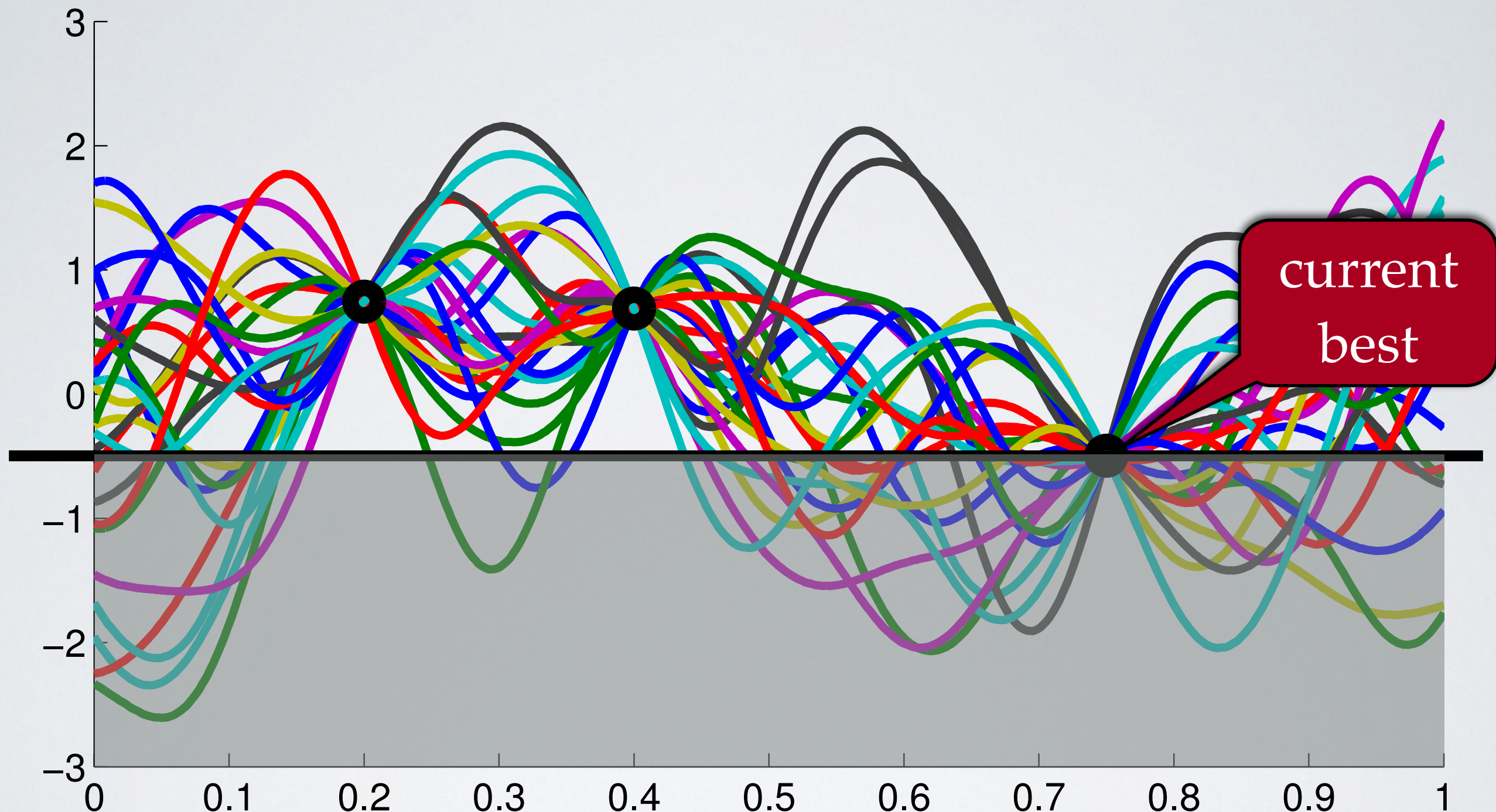
# The Bayesian Optimization Idea



Where should we evaluate next  
in order to improve the most?

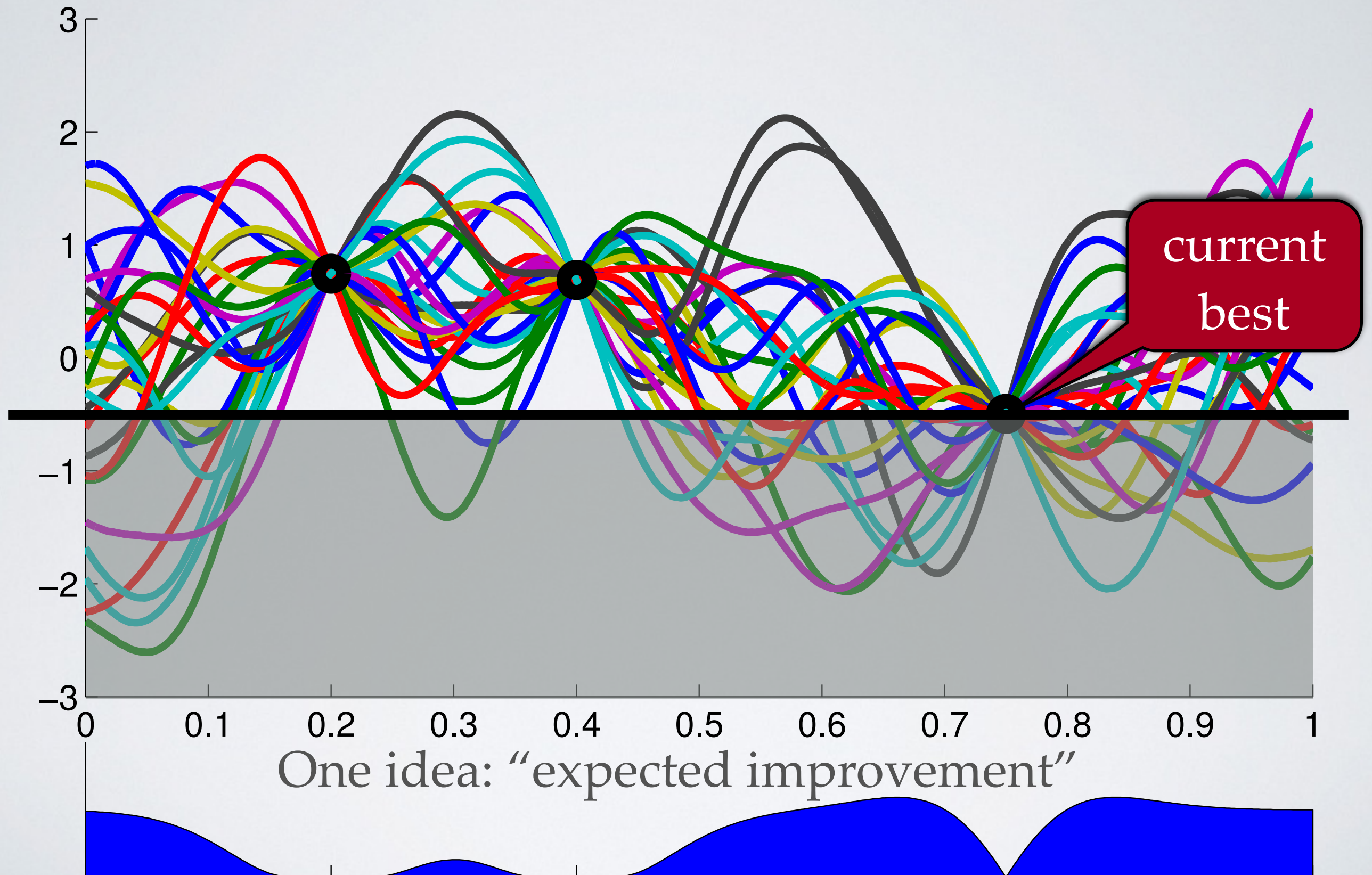


# The Bayesian Optimization Idea



Where should we evaluate next  
in order to improve the most?

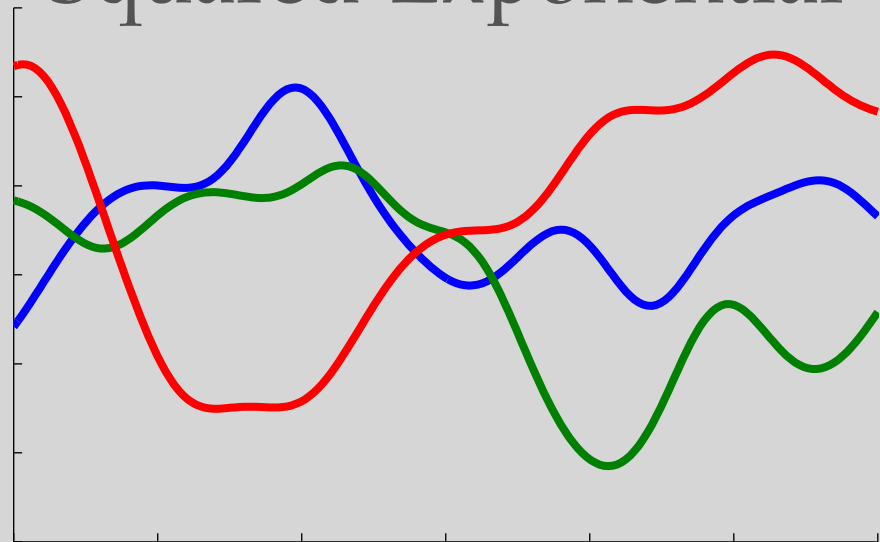
# The Bayesian Optimization Idea





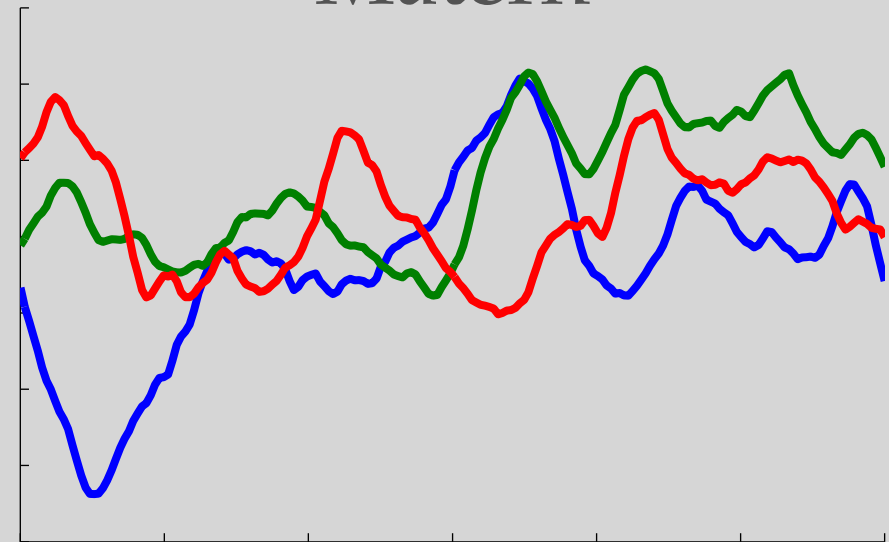
# Examples of GP Covariances

## Squared-Exponential



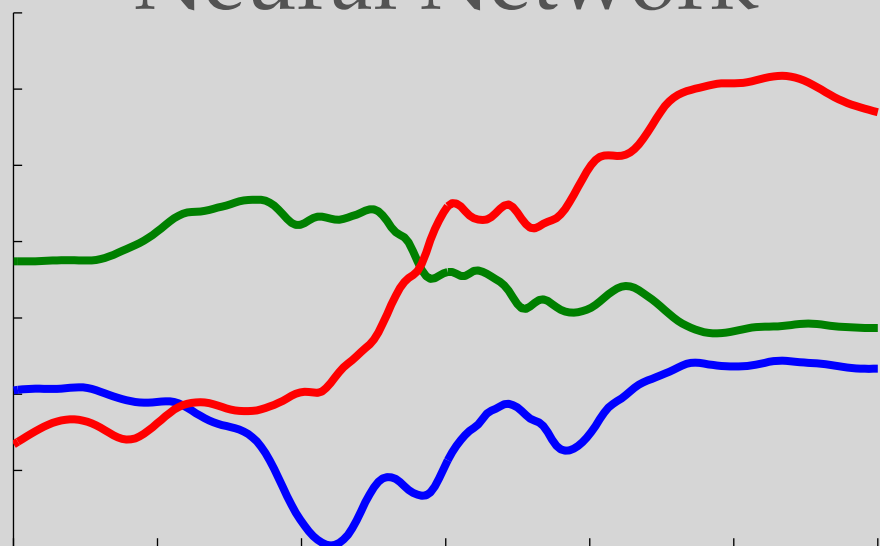
$$C(x, x') = \alpha \exp \left\{ -\frac{1}{2} \sum_{d=1}^D \left( \frac{x_d - x'_d}{\ell_d} \right)^2 \right\}$$

## Matérn



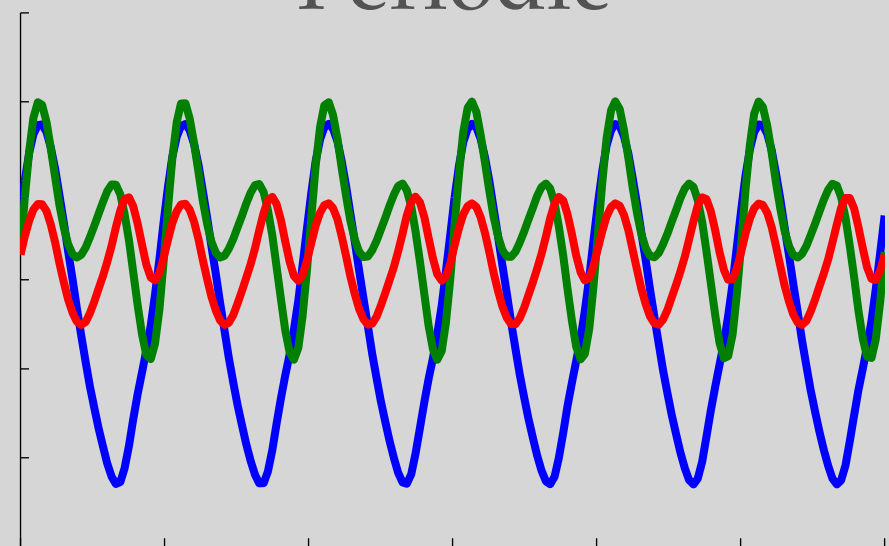
$$C(r) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left( \frac{\sqrt{2\nu} r}{\ell} \right)^\nu K_\nu \left( \frac{\sqrt{2\nu} r}{\ell} \right)$$

## “Neural Network”



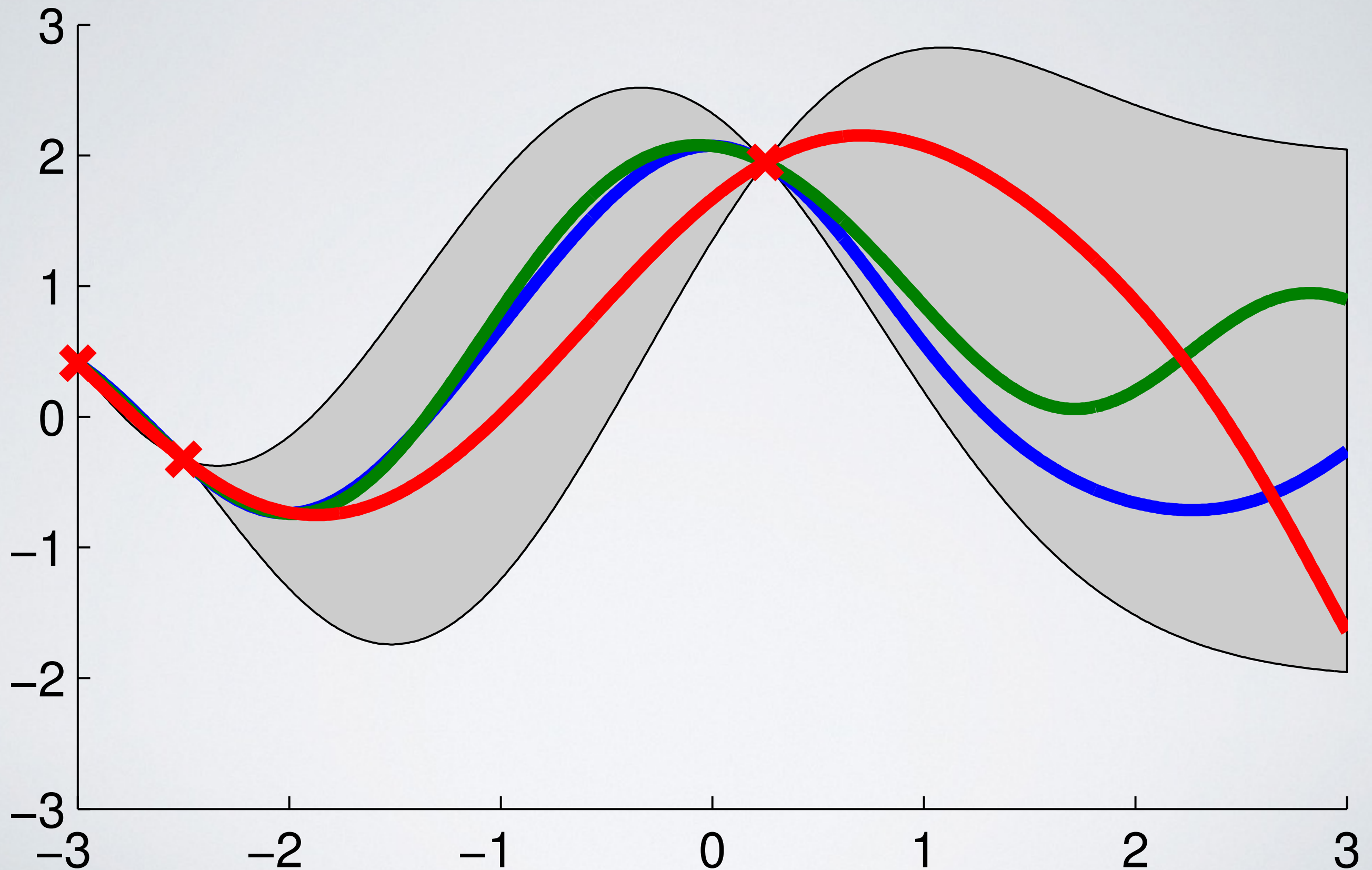
$$C(x, x') = \frac{2}{\pi} \sin^{-1} \left\{ \frac{2x^\top \Sigma x'}{\sqrt{(1 + 2x^\top \Sigma x)(1 + 2x'^\top \Sigma x')}} \right\}$$

## Periodic

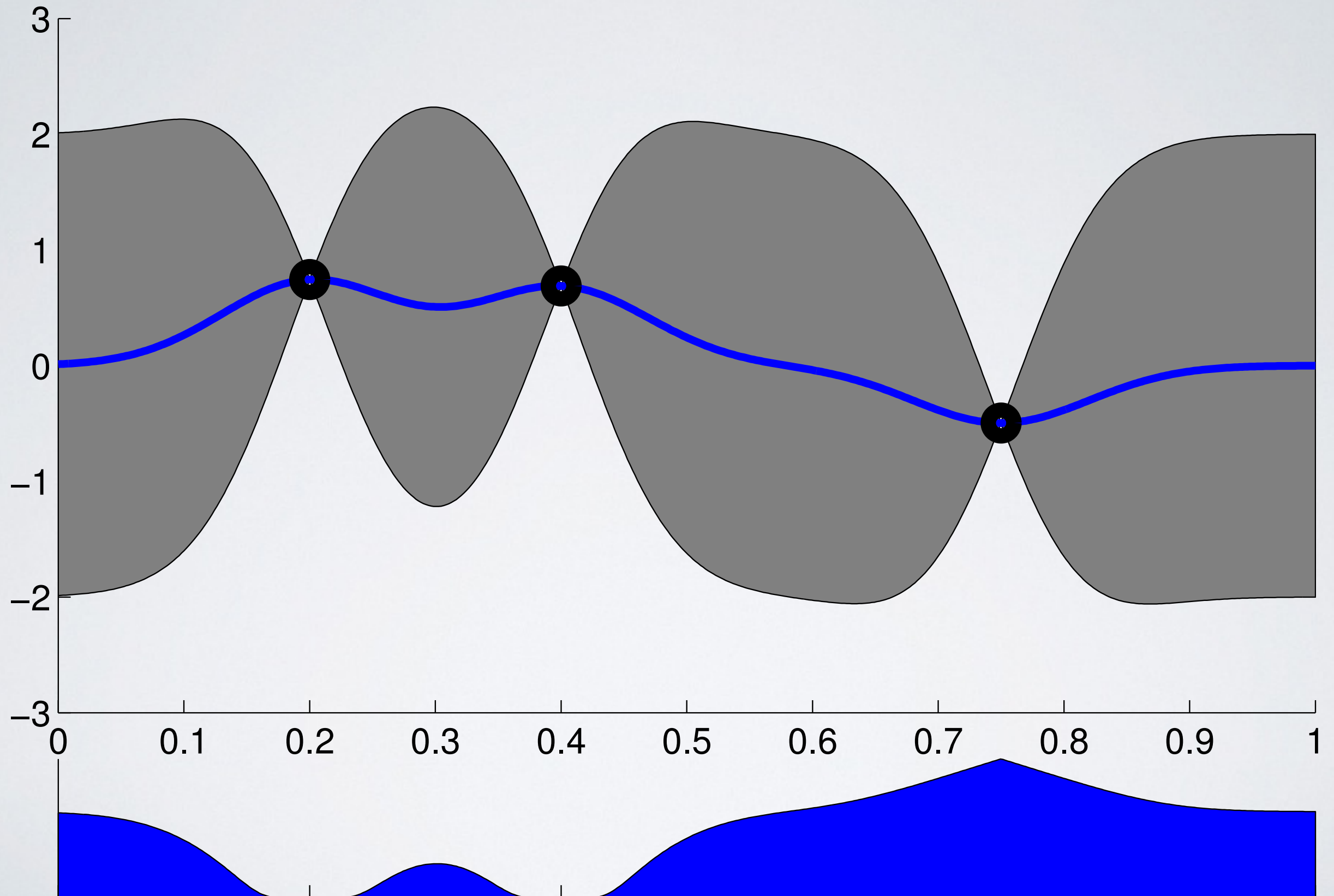


$$C(x, x') = \exp \left\{ -\frac{2 \sin^2 \left( \frac{1}{2} (x - x') \right)}{\ell^2} \right\}$$

# GPs Provide Closed-Form Predictions

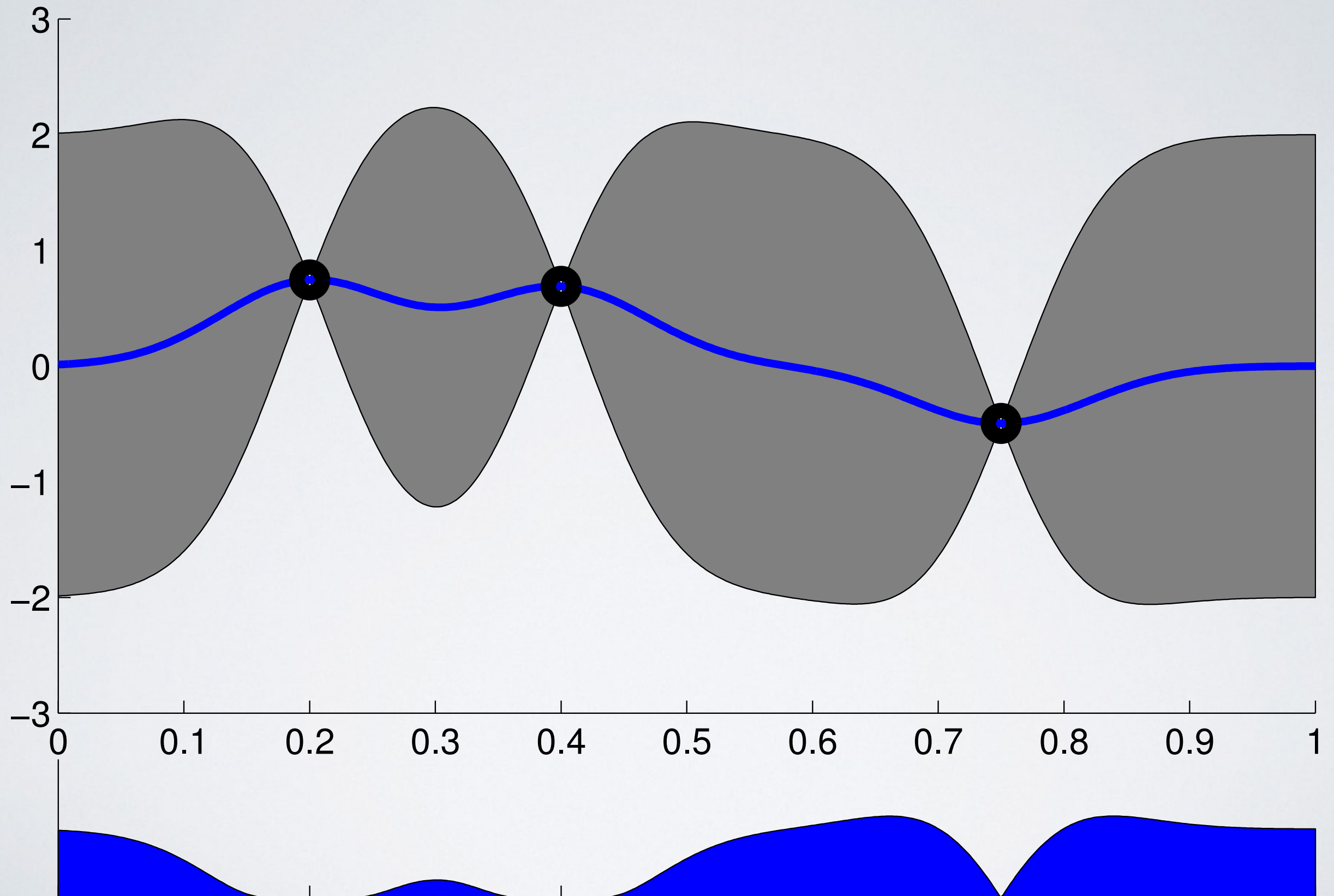


# Probability of Improvement



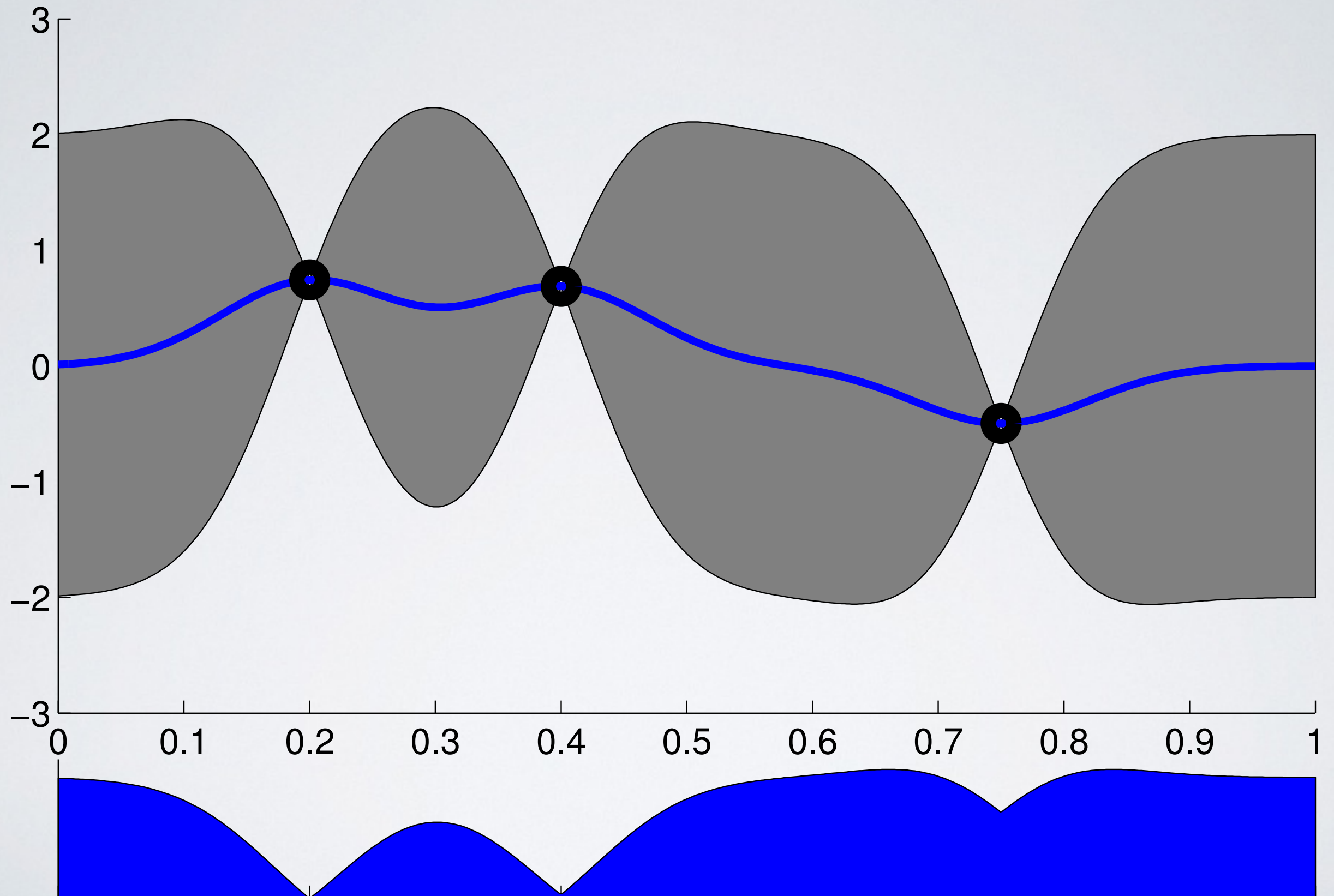
# Expected Improvement

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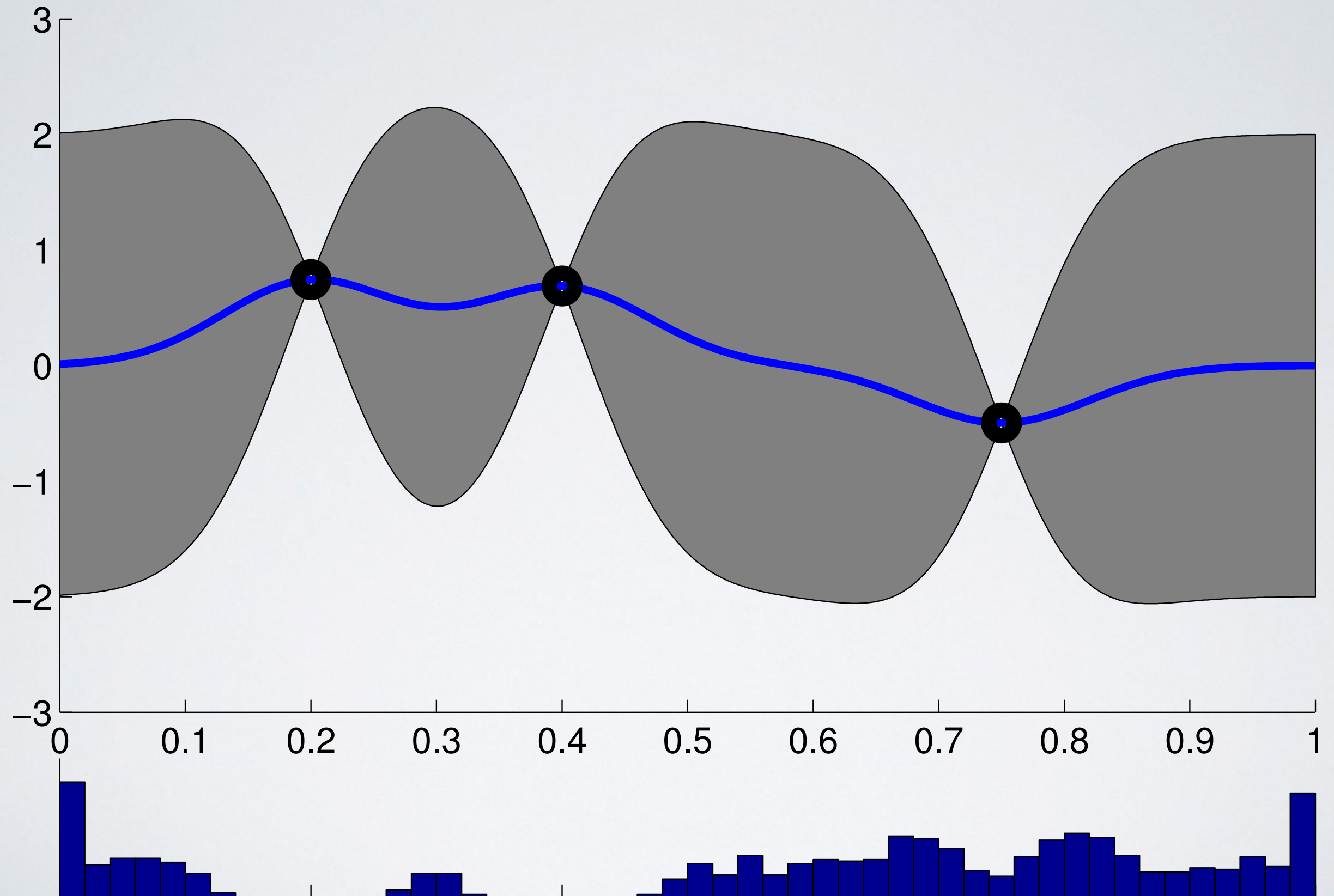
# GP Upper (Lower) Confidence Bound

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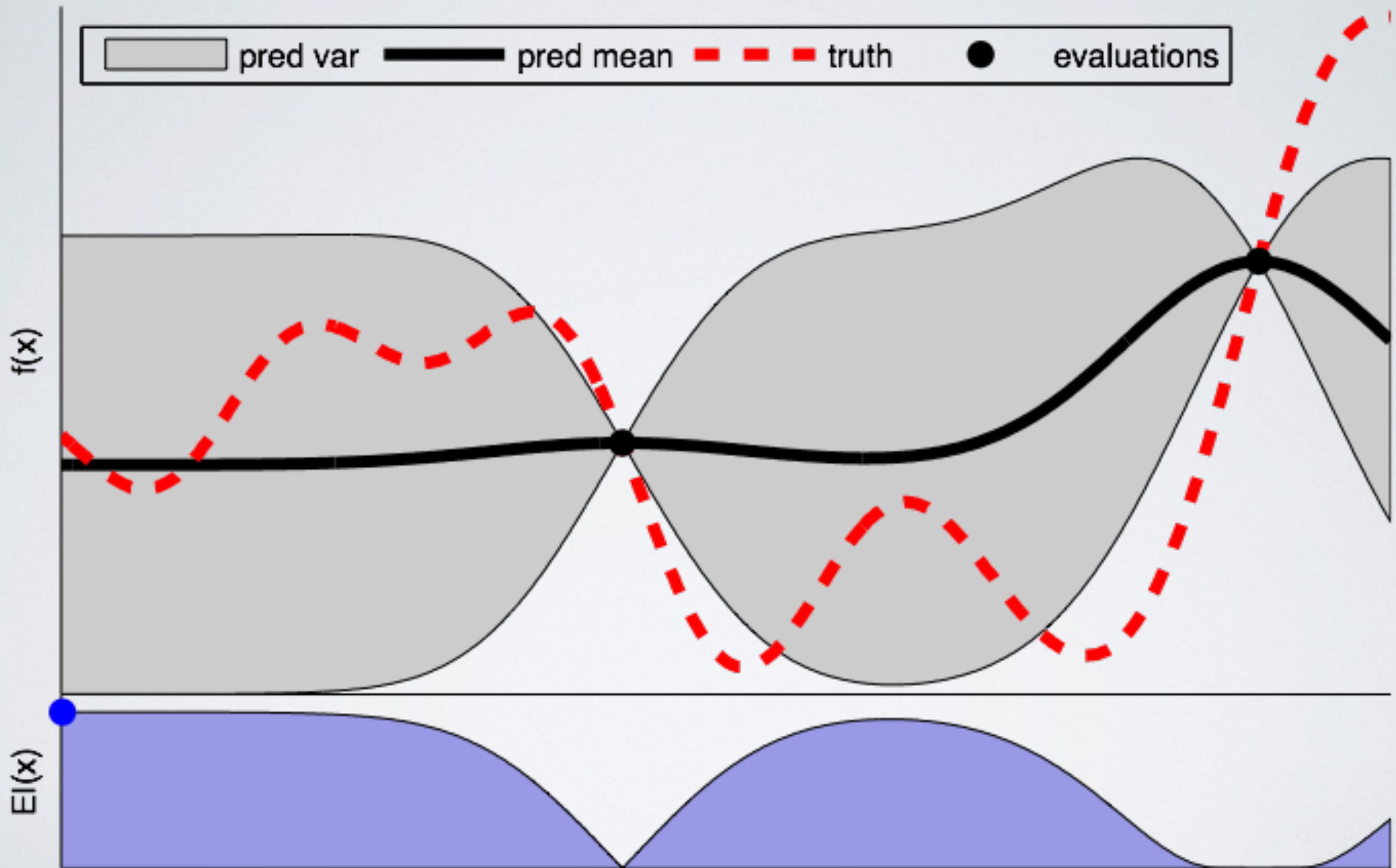




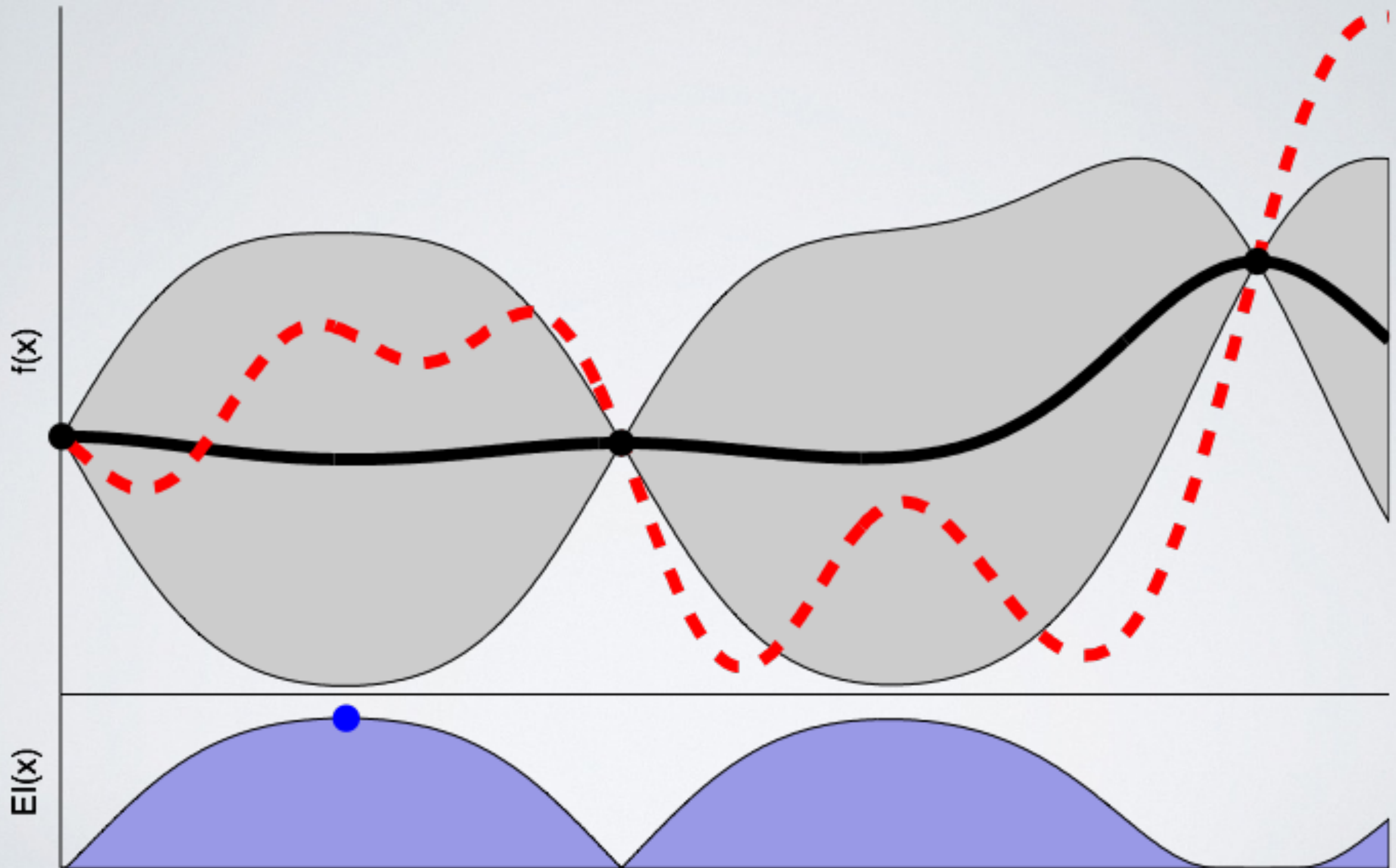
# Distribution Over Minimum (Entropy Search)



# Illustrating Bayesian Optimization

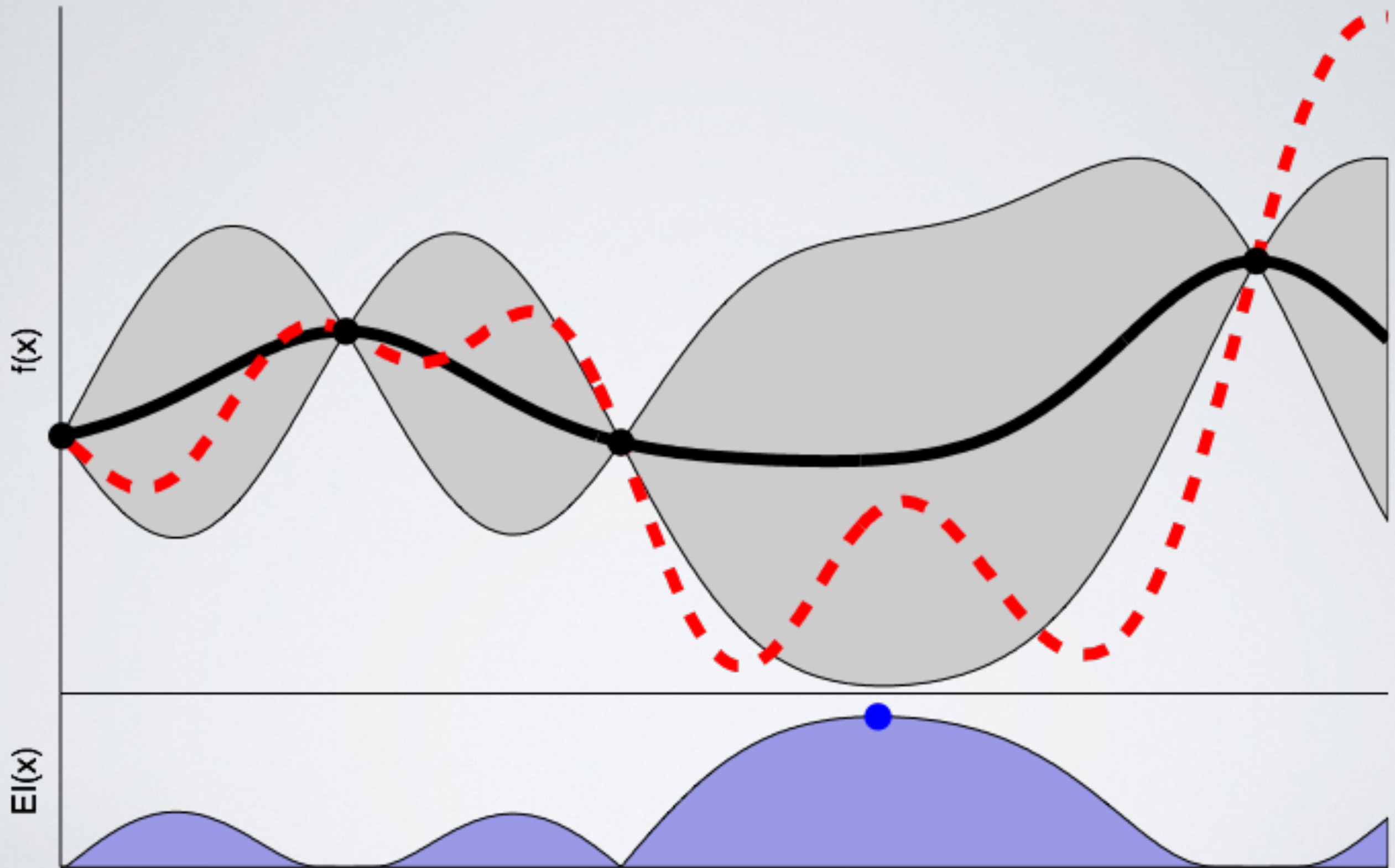


# Illustrating Bayesian Optimization

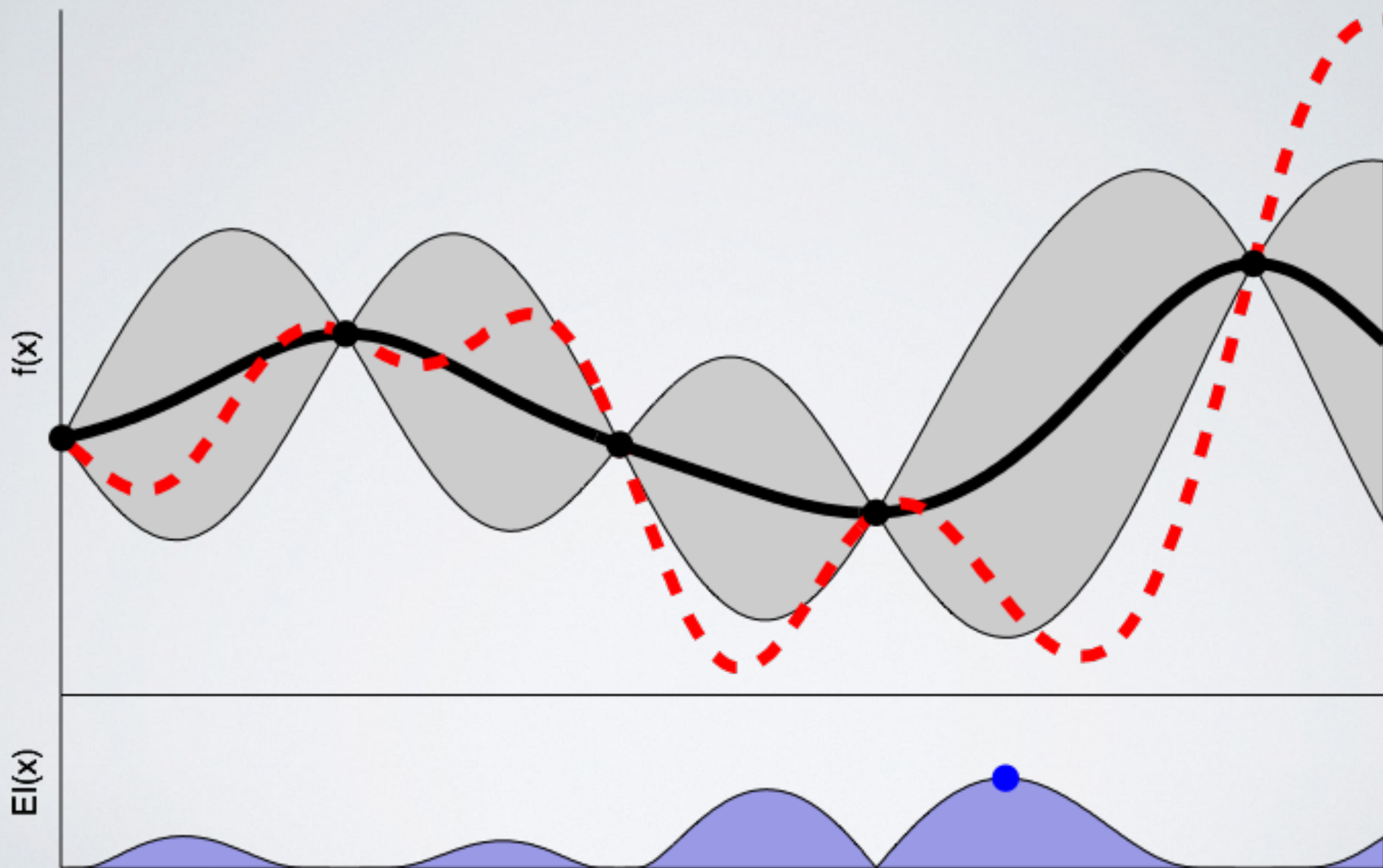




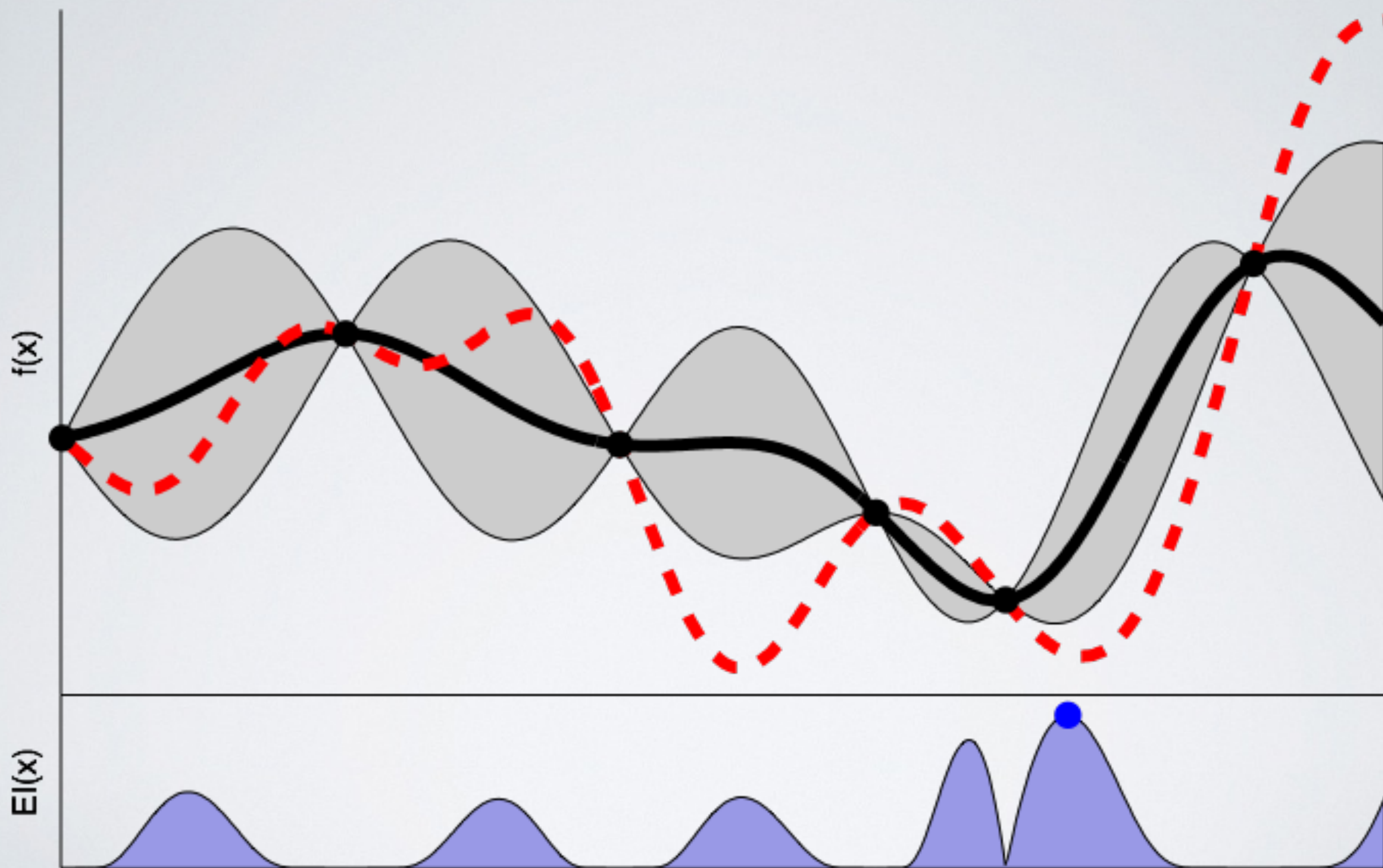
# Illustrating Bayesian Optimization



# Illustrating Bayesian Optimization

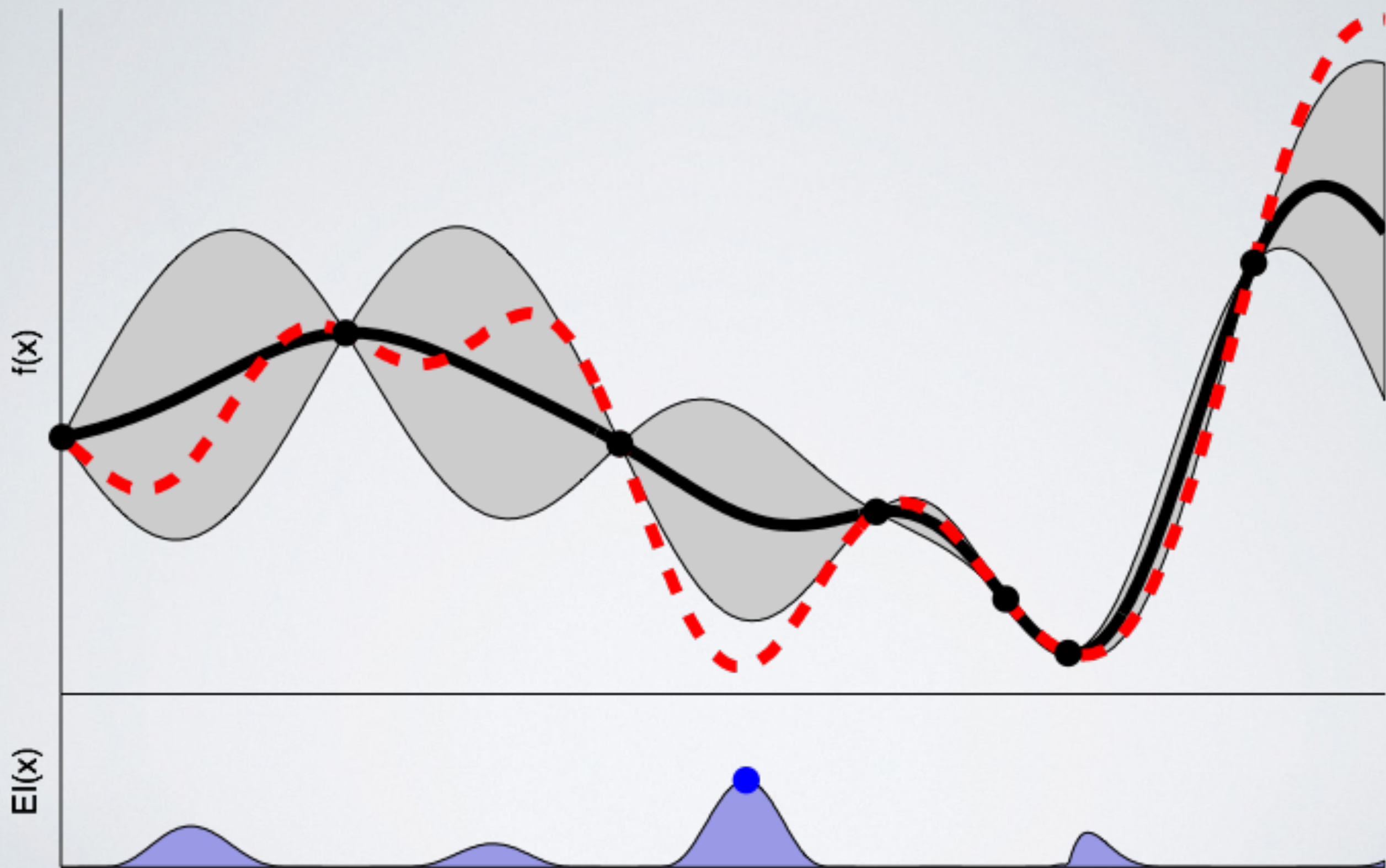


# Illustrating Bayesian Optimization

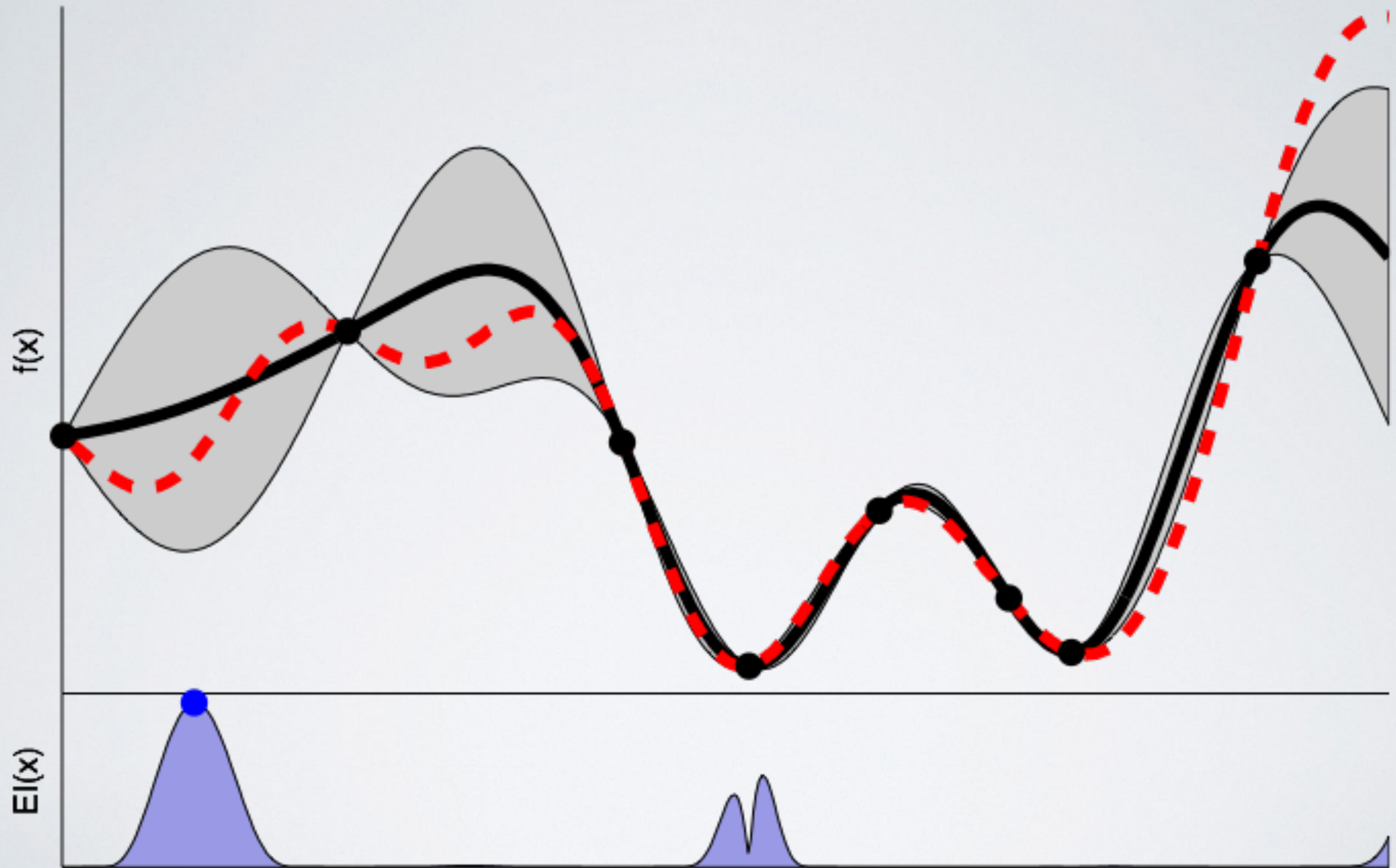




# Illustrating Bayesian Optimization



# Illustrating Bayesian Optimization



# Why Doesn't Everyone Use This?

---

These ideas have been around for *decades*.

Why is Bayesian optimization in broader use?

- ▶ **Fragility and poor default choices.**

Getting the function model wrong can be catastrophic.

- ▶ **There hasn't been standard software available.**

It's a bit tricky to build such a system from scratch.

- ▶ **Experiments are run sequentially.**

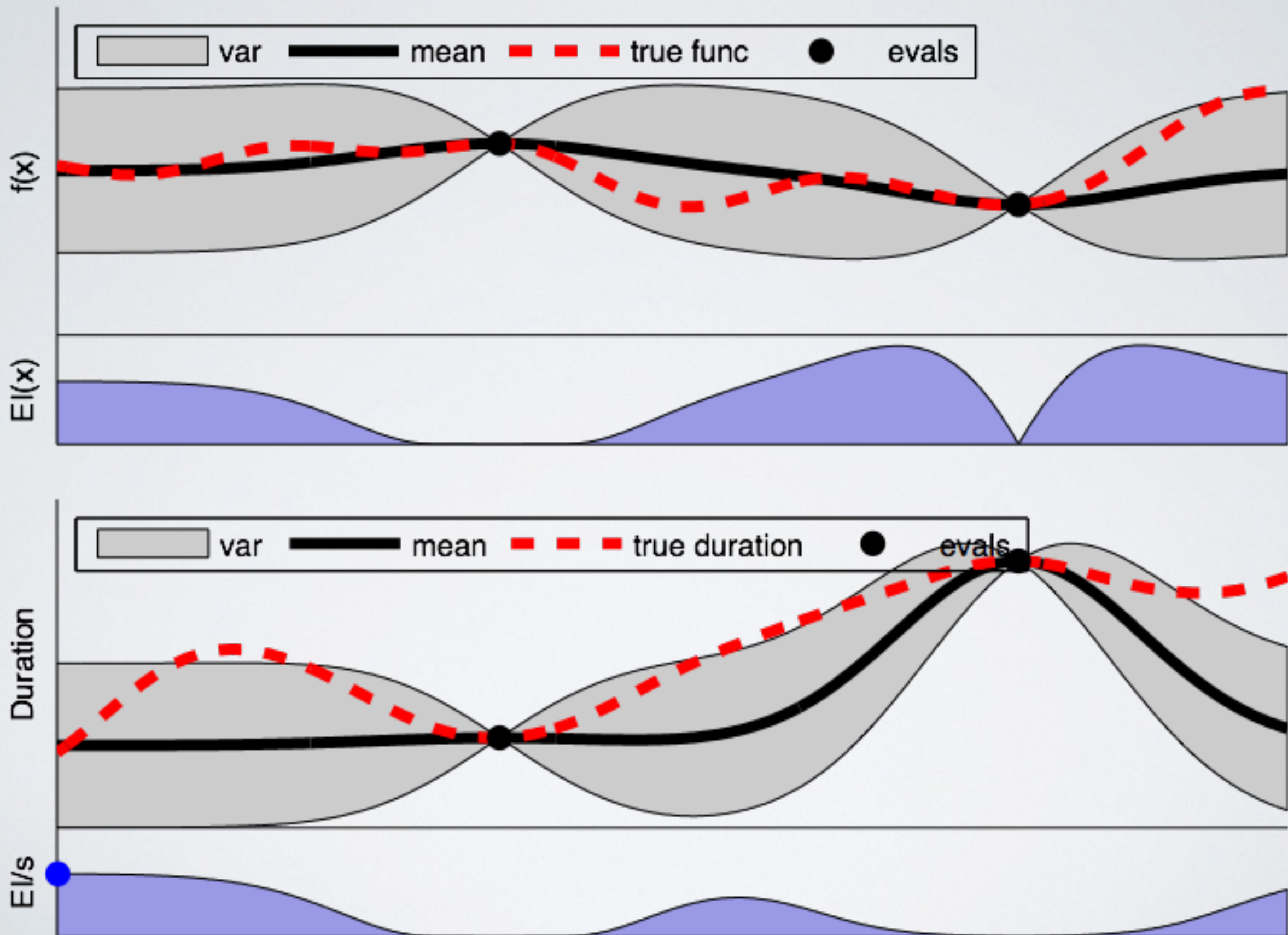
We want to take advantage of cluster computing.

- ▶ **Limited scalability in dimensions and evaluations.**

We want to solve big problems.

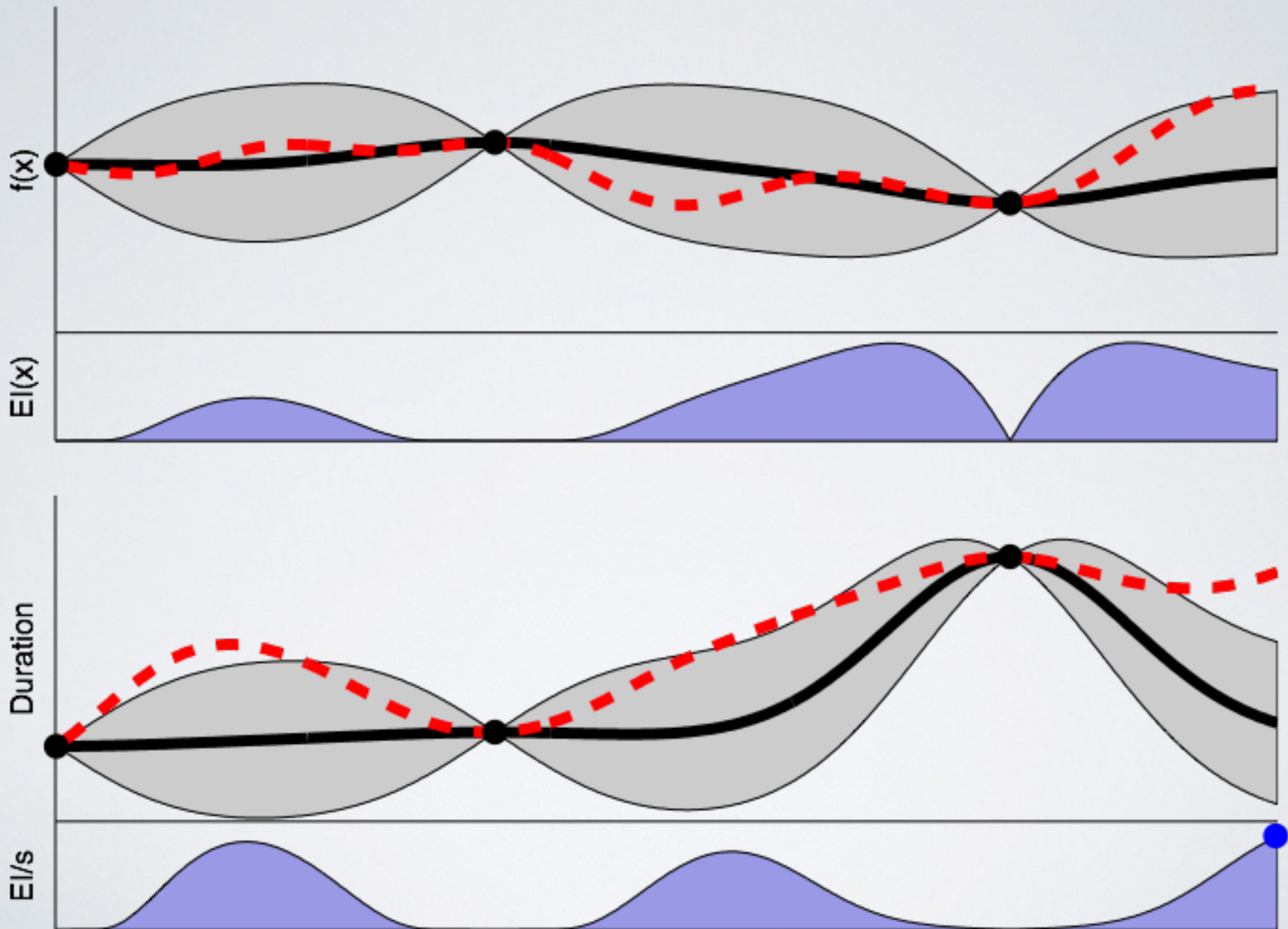


# Expected Improvement per Second

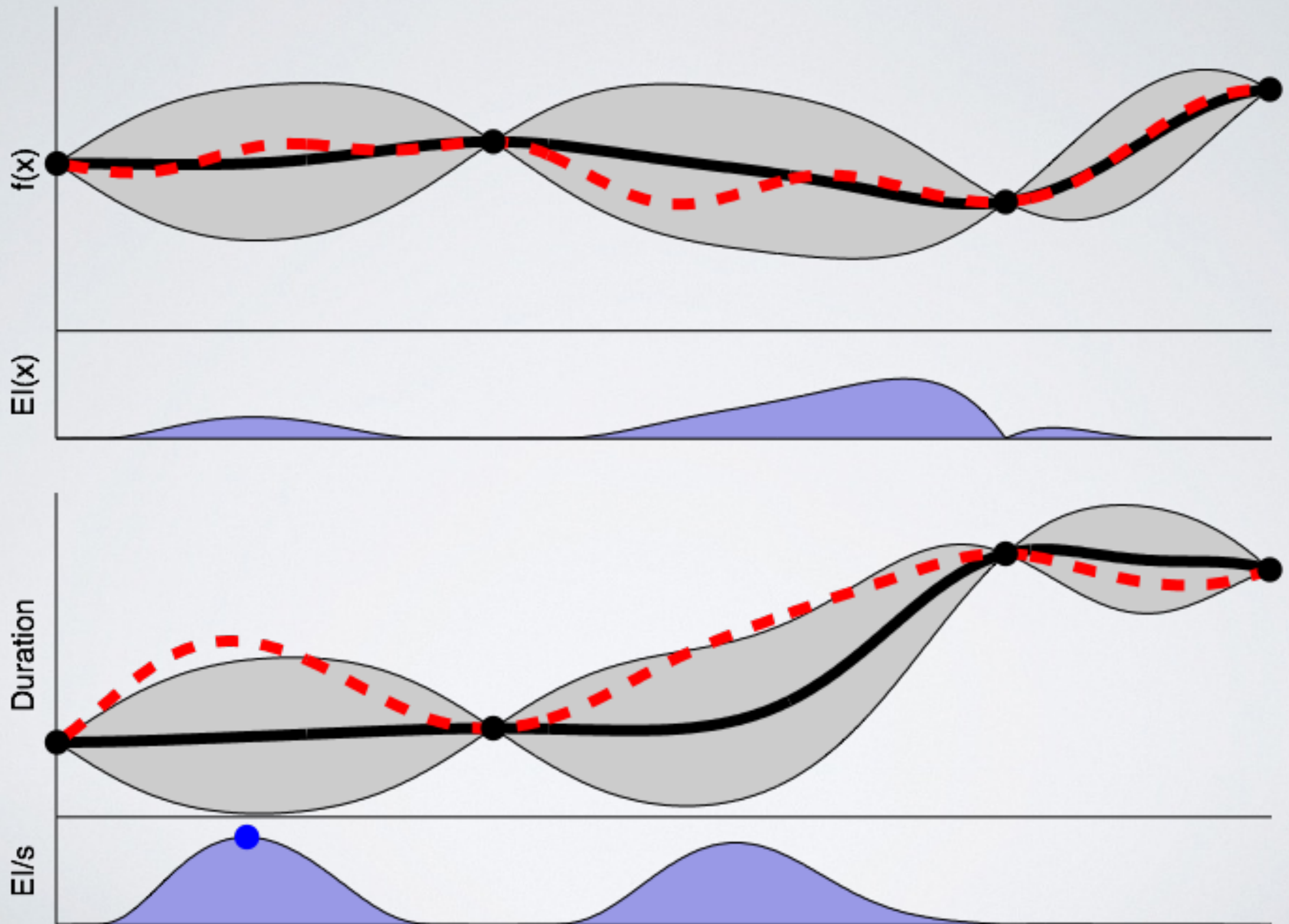




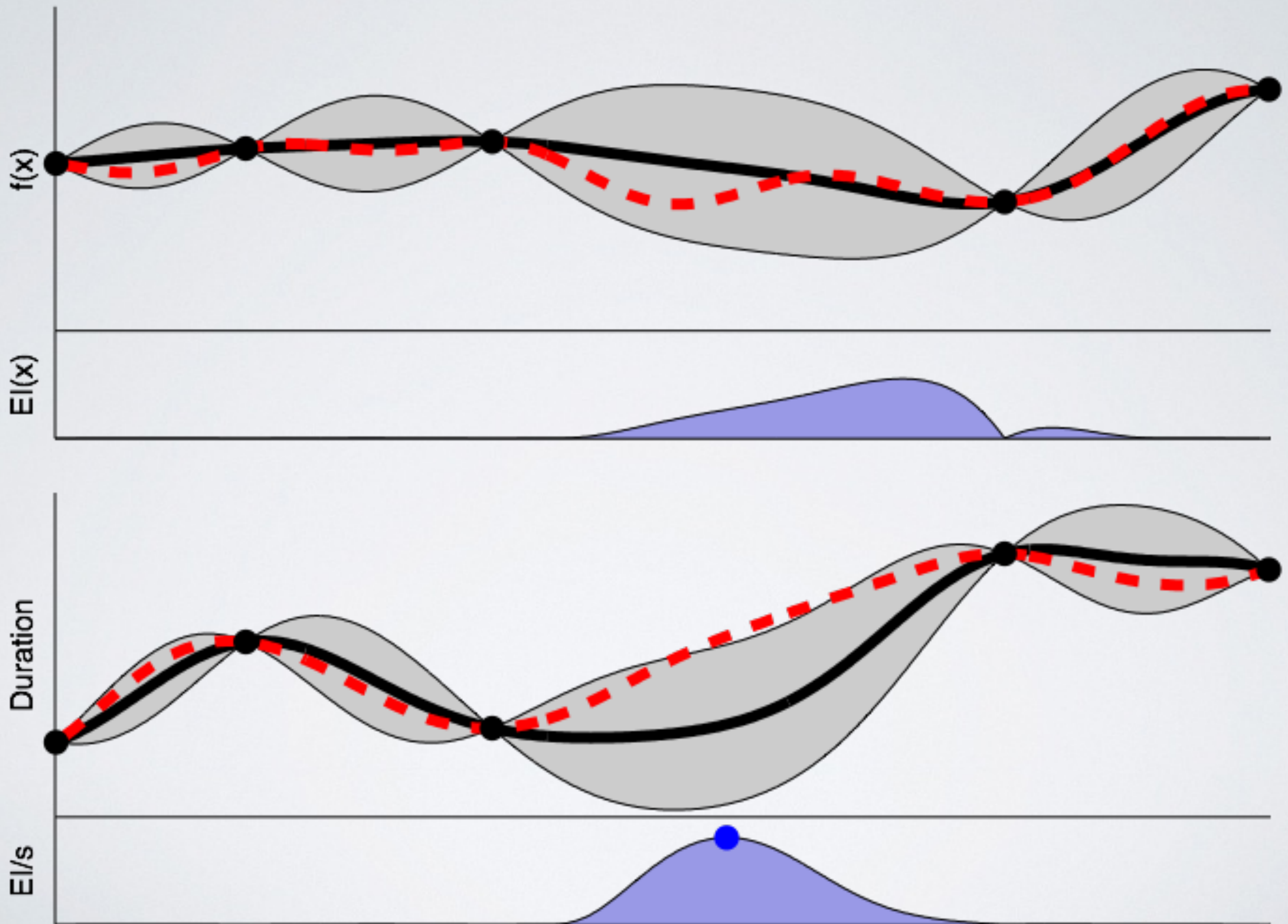
# Expected Improvement per Second



# Expected Improvement per Second

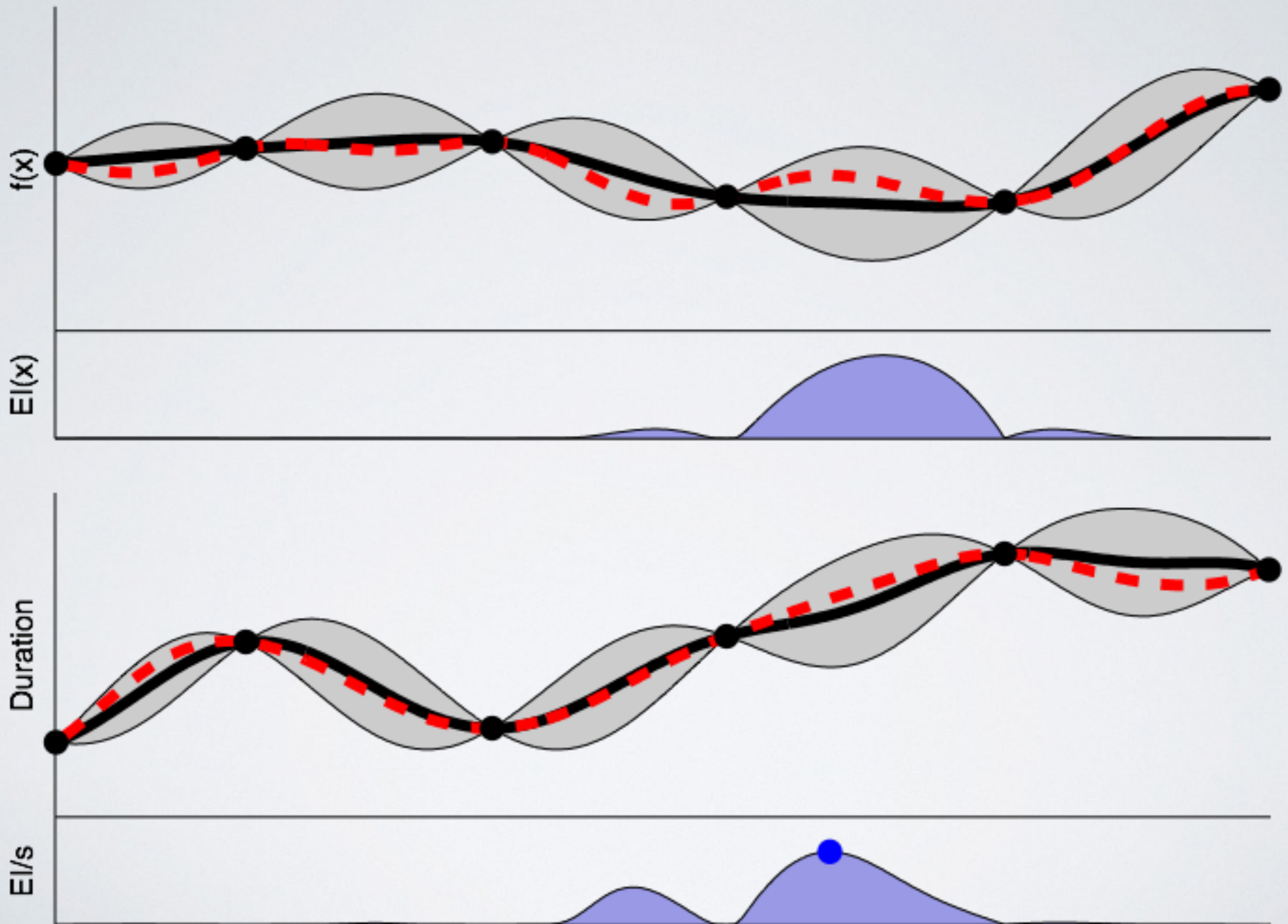


# Expected Improvement per Second



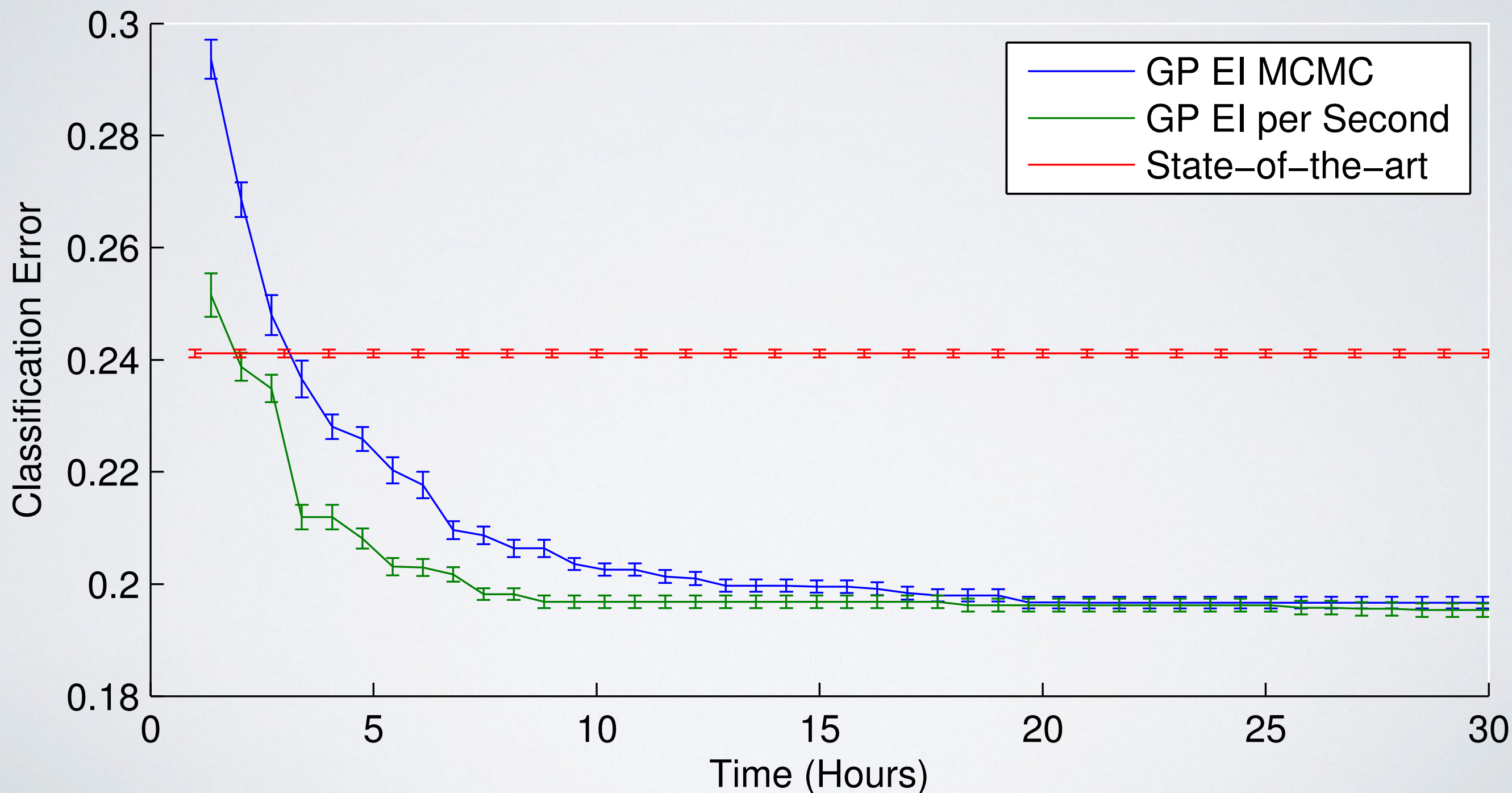


# Expected Improvement per Second



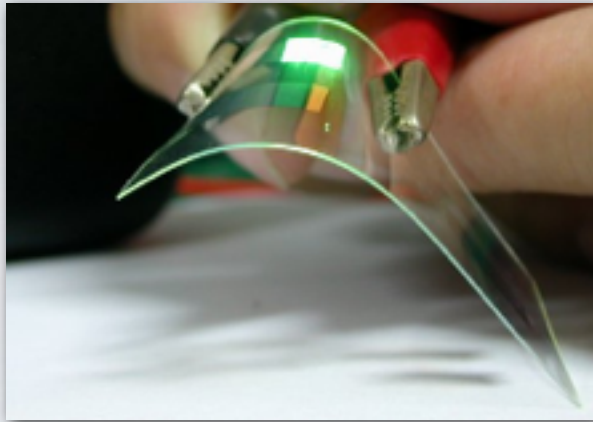
# Expected Improvement per Second

CIFAR10: Deep convolutional neural net (Krizhevsky)  
Achieves 9.5% test error vs. 11% with hand tuning.

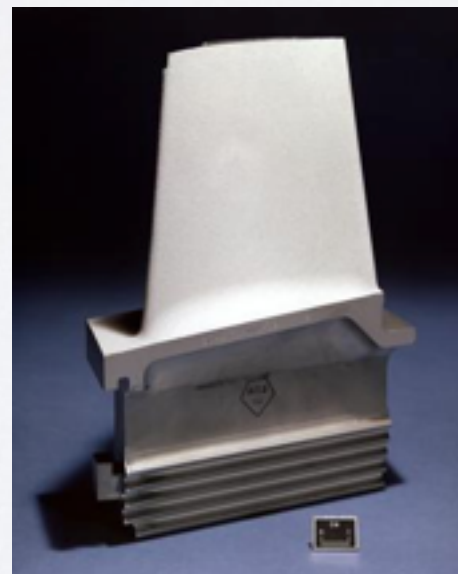




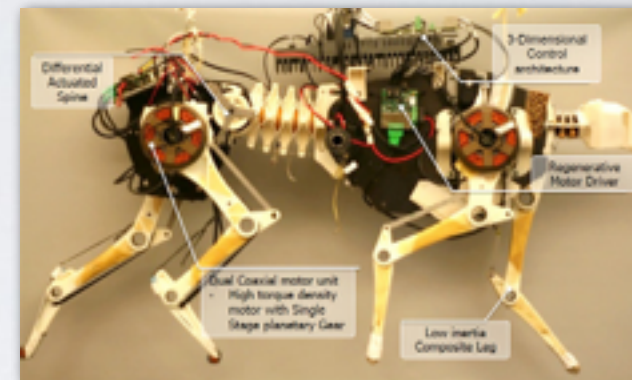
# New Directions for Bayesian Optimization



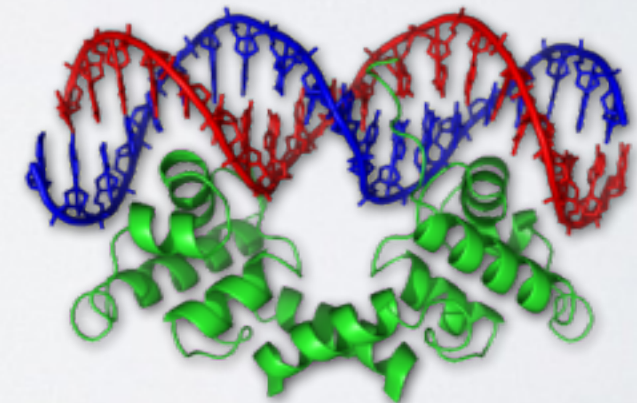
Finding new  
organic materials



Improving turbine  
blade design



Optimizing robot  
control systems



Designing DNA for  
protein affinities

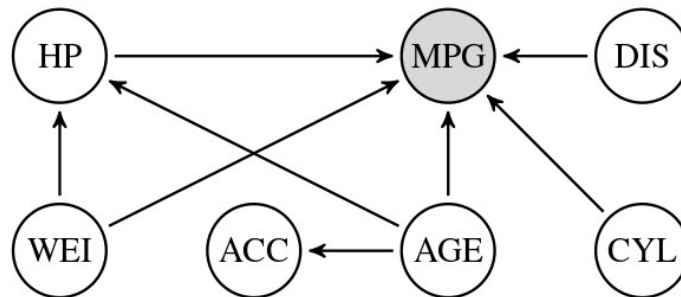


# Challenges and Perspectives

- Applying statistical methods to the global design process of a large engineering project is very difficult.
- We can attack many very useful smaller problems.
- Culture change: a point estimate shouldn't be a valid answer anymore!

# The Future of ML in Engineering?

- Probabilistic Programming for model-based ML
  1. Write down model with prior knowledge (e.g. science).
  2. Run *automated inference engine*.
  3. Analyse results, improve model and iterate.
- Causality



# Thank You!

[roger@rogerfrigola.com](mailto:roger@rogerfrigola.com)